(c) Apply Runge Kutta method of fourth order to find the approximate value of y for $x = 0.2$, in steps of 0.1, if $\frac{dy}{dx} = x + y^2$, given that y(0) = 1.

[This question paper contains 8 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 4508 **I**

Unique Paper Code : 2354000006

Name of the Paper : Numerical Methods (GE)

Name of the Course : **COMMON PROG GROUP**

Semester : V

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll. No. on the top immediately on receipt of this question paper.
2. Attempt all question by selecting two parts from each question.
3. Each question carry equal marks.
4. Use of non-programmable Scientific Calculator allowed.

1. (a) Round off the number 56468 to four significant digits and then calculate the absolute, relative and percentage errors.

(b) Find the relative error of the number 6.6 if both of its digits are correct.

(c) Determine the number of significant digits in the following numbers;

(i) 72.00,

(ii) 0.003040,

(iii) 4.0065,

(iv) 34560

x	7.47	7.48	7.49	7.50	7.51	7.52	7.53
$f(x)$	.193	.195	.198	.201	.203	.206	.208

(c) Evaluate $\int_0^1 \frac{1}{1+x^2} dx$ by using Trapezoidal rule. Hence obtain the approximate value of π.

6. (a). Given $\frac{dy}{dx} = \frac{y-x}{y+x}$ with the initial condition x = 0, y = 1. Find y(0.1) using Euler's method.

(b) Using mid-point (Runge Kuttą second order) method, solve the initial value problem $\frac{dy}{dx} = x^2 + \sqrt{y}$ from $x = 0$ *to* $x = 0.6$ where $y(0) = 1$ by using $h = 0.2$.

5. (a) Use the three point backward difference formula

$$f'(x_i) \approx \frac{3f(x_i) - 4f(x_i - h) + f(x_i - 2h)}{2h}$$

to approximate the derivative of $f(x) = \sin x + \cos x$ at $x_i = \frac{\pi}{2}$, taking $h = 1$, 0.1 and 0.01.

(b) For the following data, find $f''(7.50)$ by using the formula

$$f''(x_i) \approx \frac{f(x_i) - 2f(x_i + h) + f(x_i + 2h)}{h^2}$$

(v) 1.02×10^{-4}

2. (a) Perform five iterations of the bisection method to obtain the smallest positive root of the equation $f(x) = x^3 - 5x + 1 = 0$, also write the order of convergence of Bisection method.

(b) Using Regula Falsi method compute the real root of the equation $x^2 = 2$. Correct to four decimal places, also write the order of convergence of Regula Falsi method.

(c) Using Newton-Raphson Method compute $\sqrt{19}$ correct to four decimal places, also write the order of convergence of Newton-Raphson Method.

(d) Using Secant method, find the smallest positive root of the equation $x^3 + 3x^2 = 1$ correct to three

decimal digits, also write the order of convergence of Secant Method.

3. (a) Using Gauss elimination method solve the following system of linear equations

$$x+4y+z=1$$
$$2x+2y+z=2$$
$$3x+6y+z=3$$

(b) Show that $E^3 = \Delta^3 + 3E^2 - 3E + 1$ (Note: Symbols have their own meaning)

(c) Given $f(x) = \log(1+x)$, $x_0 = 1$ and $x_1 = 1.3$.

Use Lagrange interpolation to calculate approximate value of $f(1.04)$.

4. (a) Starting with initial vector $(x, y, z) = (0,0,0)$ perform three iterations of Gauss-Seidel method to solve the following system of equations

$$10x-2y+z=9$$
$$-2x+10y-2z=12$$
$$-2x-5y+10z=18$$

(b) Find the unique polynomial P(x) of degree 2 or less using Newton's interpolating formula for the following data

x	0	1	2	3
f(x)	1	2	4	8

Also, estimate $P(1.5)$.

(c) Calculate the 3rd divided difference of $\frac{1}{x}$, based on the points x_0, x_1, x_2, x_3.

P.T.O.