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Unique Paper Code	: 2224001002
Name of the Paper	: Mathematical Physics (GE paper)
Name of the Course	: B.Sc. Hons.-Physics_NEP: UGCF-2022
Semester	: I-Semester
Duration	: 3 Hours
Maximum Marks	: 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt five questions in all.
3. Question No. 1 is compulsory.
4. From Question No. 1 attempt only 6 questions.

1. (6X3=18)

(a) Find the Fourier series expansion for

$$f(x) = |x| \qquad -2 < x < 2$$

(b) Find the value of $\Gamma\left(\frac{7}{2}\right)$.

(c) Prove the relation between gamma and beta function.

(d) Show that $e^{-\left(\frac{1}{z^2}\right)}$ has no singularities.

(e) Find the regular singular point of the given differential equation

$$x^2(x-2)^2\frac{d^2y}{dx^2} + 2(x-2)\frac{dy}{dx} + (x+3)y = 0$$

(f) Check whether $f(z) = \sin z$ is analytic or not.

(g) If the real part of a complex analytic function is $u(x,y) = x + \frac{1}{2}(x^2 - y^2)$, find the corresponding imaginary part.

2.

(a) Given that $f(x) = x + x^2$ for $-\pi < x < \pi$, find the fourier expression of $f(x)$.

Deduce that (10)

$$\frac{\pi^2}{6} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \ldots\ldots\ldots\ldots\ldots$$

(b) Prove that (4)

$$\beta(m, n) = \beta(n, m)$$

(a) Prove (4)

$$\Gamma(n+1) = n!$$

3.

(a) Consider the following differential equation (10)

$$x(x-1)\frac{d^2y}{dx^2} + (3x-1)\frac{dy}{dx} + y = 0$$

Using Frobenius method, determine the roots of indicial equation and hence find the first solution.

(b) Also, find the second solution. (4)

(c) Evaluate (4)

$$\int_0^1 (1-x^3)^{-\frac{1}{2}} dx$$

4.

(a) Prove that (6)

$$J_{-\frac{1}{2}}(x) = \sqrt{\left(\frac{2}{\pi x}\right)}\cos x$$

(b) Prove that (6)

$$\int_0^\infty e^{-u^2} du = \frac{\sqrt{\pi}}{2}$$

(c) Evaluate the following integral using residue theorem (6)

$$\int_c \frac{1+z}{z(2-z)}dz$$

where, c is the circle $|z| = 1$.

5.

(a) Solve (10)

$$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = e^{3x} + 5e^{-2x} - \log 2$$

(b) Determine whether or not $u(x,y) = 4e^{-3x}\cos 3y$ is a solution of given differential equation: (4)

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

(c) Prove that (4)

$$J_{-n}(x) = (-1)^n J_n(x)$$

6.

(a) Apply calculus of residues to show that (8)

$$\int_0^\infty \frac{dx}{(x^2+1)(x^2+9)} = \frac{\pi}{24}$$

(b) If $z(x+y) = (x^2+y^2)$ (6)

Find

$$\frac{\partial z}{\partial x} \text{ and } \frac{\partial z}{\partial y}$$

and show

$$\left(\frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right)^2 = 4\left(1 - \frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right)$$

(c) Show (4)

$$\Gamma(n)\Gamma(n-1) = \frac{\pi}{\sin n\pi}$$