6. (a) If α, β and γ are the roots of the equation $x^3 + 2x^2 - 3x - 1 = 0$, then find the value of $\alpha^{-3} + \beta^{-3} + \gamma^{-3}$.

(b) If α, β and γ are the roots of the equation $x^3 + 2x^2 - x + 1 = 0$, find a cubic equation with the roots $\frac{2}{\alpha}, \frac{2}{\beta}, \frac{2}{\gamma}$.

(c) Find s_2 and s_4 for the equation $x^3 + 2x^2 - 3x + 4 = 0$.

(7½,7½,7½)

[This question paper contains 4 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 3554 **I**

Unique Paper Code : 2354001002

Name of the Paper : Theory of Equations and Symmetries

Name of the Course : **GE**

Semester : I

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **all** questions by selecting **two** parts from each question.
3. Parts of the question to be attempted together.
4. Each part carries **7.5** marks.
5. Use of Calculator is not allowed.

1. (a) Find the quotient and remainder when $3x^4 - 5x^3 + 10x^2 + 11x - 61$ is divided by $x - 3$.

(b) Solve the equation $x^3 - 3x^2 - 6x - 20 = 0$, one root is given as $-1 + \sqrt{-3}$.

(c) State Factor theorem for a polynomial. Find a quartic equation having two double roots 3 and -3. (7½,7½,7½)

2. (a) Find the fourth root of the complex number $1 + i$.

(b) Find the equation whose roots are diminished by 3 than the roots of $x^4 - 7x^3 + 3x^2 - 11x + 17 = 0$.

(c) Find the equation whose roots are the roots of the equation $x^5 + 7x^4 + 7x^3 - 8x^2 + x + 1 = 0$ with their signs changed. (7½,7½,7½)

3. (a) Prove that $1 + \omega + \omega^2 = 0$, where ω is imaginary cube root of unity.

(b) Using Descartes rule of sign, find the least number of imaginary roots of the equation

$$x^6 - 3x^2 - x + 1 = 0.$$

(c) Transform the equation $x^3 - 6x^2 + 4x - 7 = 0$ into one which does not contain the third term. (7½,7½,7½)

4. (a) Solve the equation $x^3 - 15x - 126 = 0$ using Cardan's method.

(b) Multiplying the roots by a suitable number transform the equation $x^4 + \frac{3}{10}x^2 \frac{13}{25}x + \frac{77}{1000} = 0$, into one with integral coefficients and leading coefficient unity.

(c) Find the condition that the cubic equation $x^3 - px^2 + qx - r = 0$ has its roots in geometric progression. (7½,7½,7½)

5. (a) If α, β and γ are the roots of the equation $x^3 + px^2 + qx + r = 0$, then prove that

$$\Sigma\alpha\,\Sigma\alpha\beta = \Sigma\alpha^2\beta + 3\alpha\beta\gamma, \text{ whence } \Sigma\alpha^2\beta = 3r - pq.$$

(b) Find an equation whose roots are 3-times the roots of the equation $x^4 - 3x^3 + 5x - 1 = 0$.

(c) If α, β, γ and δ are the roots of the equation $x^4 + px^3 + qx^2 + rx + s = 0$, prove that

$$\sum\frac{\alpha^2+\beta^2}{\alpha\beta} = \sum\frac{\beta}{\alpha}.$$ (7½,7½,7½)