(ii) Find the interval in which the function is concave up or concave down.

(b) Determine the critical points of the function $f(x) = \frac{(x+1)^2}{x^2+1}$. Also, find the horizontal and vertical asymptotes.

(c) Trace the graph of the function given by

$$f(x) = x^4 - 4x^3 + 10.$$

6. (a) Determine the intervals where the function given by $f(x) = 6x^{\frac{1}{3}} + 3x^{\frac{4}{3}}$ is concave up or concave down. Also, find the points of inflection, if any.

(b) Find the critical points of $f(x) = x^{\frac{1}{3}}(x-4)$. Find the intervals on which f is increasing or decreasing. Also, find the local extreme values of the function.

(c) Find the oblique asymptotes of the curve given by the equation

$$4x^3 - x^2y - 4xy^2 + y^3 + 3x^2 - y^2 + 2xy = 7$$

[This question paper contains 4 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 3553 **I**

Unique Paper Code : 2354001001

Name of the Paper : GE : Fundamentals of Calculus

Name of the Course : **Common Prog. Group**

Semester : I

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** questions are compulsory and carry equal marks.
3. This question paper has **six** questions.
4. Attempt any **two** parts from each question.
5. Use of Calculator not allowed.

1. (a) Evaluate the following limits.

(i) $\lim_{x \to 0} x^2 \sin \frac{1}{x^2}$

(ii) $\lim_{x \to 0} \frac{3 \sin^{-1} x}{4x}$

(b) Examine the continuity and differentiability of the function

$$f(x) = \begin{cases} x^2 \cos\frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

(c) Find the n^{th} derivative of $y = \tan^{-1} x$.

2. (a) If $y = \left(x + \sqrt{1+x^2}\right)^m$, find the value of y_n at $x = 0$.

(b) If $u = f(r)$, where $r^2 = x^2 + y^2$,

show that : $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f''(r) + \frac{1}{r} f'(r)$

(c) If $u = \tan^{-1} \frac{x^3+y^3}{x-y}$, $x \neq y$,

show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u$ and

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = (1 - 4\sin^2 u) \sin 2u$$

3. (a) State Rolle's theorem.

Show that there is no real number t for which the equation $x^3 - 3x + t = 0$ has two distinct solutions in the interval $[-1, 1]$.

(b) State Lagrange's Mean value theorem. Use it to prove that

$$\frac{x-1}{x} < \log x < x - 1 \text{ for } x > 1.$$

(c) State and prove Cauchy's Mean value theorem.

4. (a) State Taylor's theorem with Lagrange's form of remainder.

Show that for any $k \in \mathbb{N}$

$$\log(1+x) < x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots + \frac{x^{2k+1}}{2k+1} \text{ for } x > 0$$

(b) Evaluate $\lim_{x \to 0} \frac{\tan^2 x - x^2}{x^2 \tan^2 x}$.

(c) For which value of constants, a and b is it true that

$$\lim_{x \to 0} (x^{-3} \sin 2x + a x^{-2} + b) = 1$$

5. (a) For the function given by $f(x) = \frac{(x^2+1)}{(x^2-9)}$.

(i) Find the intervals in which the function is increasing or decreasing.