[This question paper contains 2 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : 1550 **I**

Unique Paper Code : 2223512010

Name of the Paper : Physics DSE - 13b: Mathematical Physics I

Name of the Course : **B.Sc. (P) Physical Science**

Semester : III

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **five** questions in all. Question number **1** is compulsory.

1. Attempt any **six** parts. **All** parts carry equal marks.

(a) Use chain rule to evaluate $\frac{du}{dx}$ if $u = y\sin(x)$ and $y = x\,e^{-x}$.

(b) Determine if $(1 + 2xy^3)dx + 3x^2y^2\,dy$ is an exact differential.

(c) Show that $\frac{1}{x^2}$ is an integrating factor of differential equation $-ydx + xdy = 0$.

(d) Evaluate $\int_{-\pi}^{\pi} \sin(mx)\sin(nx)\,dx$.

(e) For the differential equation,

$$x^3(x - 1)y'' - 29x - 1)y' + 3xy = 0,$$

locate and classify its singular point(s) on the x-axis.

(f) Given $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$, evaluate $\frac{\Gamma\left(\frac{5}{2}\right)}{\Gamma\left(\frac{1}{2}\right)}$.

(g) Solve the partial differential equation

$$3\frac{\partial u}{\partial x} + 2\frac{\partial u}{\partial y} = 0$$

by separation of variable method. (3×6=18)

2. (a) Determine the Fourier series of a periodic function f(x) with period 2π given by

$$f(x) = \begin{cases} -1 & \text{for} \quad -\pi < x < 0 \\ 1 & \text{for} \quad 0 < x < \pi \end{cases}$$

P.T.O.

(b) Given $f(x) = x$ for $0 < x < L$, and $f(x + 2L) = f(x)$, write the Fourier series representation of $f(x)$ if it is an even function. (9,9)

3. (a) Find the dimensions x, y, z of a closed rectangular box of surface area S such that its volume is maximum.

(b) Show that $p(x) = e^x$ is an integrating factor for the differential equation

$$\sin(y)\, dx + \cos(y)\, dy$$

and hence solve the differential equation. (9,9)

4. (a) Show that z=0 is a regular singular point of the differential equation

$$4z\frac{d^2y}{dx^2} + 2\,\frac{dy}{dx} + y = 0.$$

(b) Use the method of Frobenius to find the series solution of Legendre's differential equation

$$(1-x^2)\frac{d^2y}{dx^2} - 2x\,\frac{dy}{dx} + n(n+1)y = 0$$ (6,12)

5. (a) Evaluate $\int_{-1}^{1} P_2(x)\, P_3(x)dx$, $P_2(x)$ and $P_3(x)$ being the Legendre polynomials of degree 2 and 3 respectively.

(b) Show that the beta function $B(x, y) = B(y, x)$. Evaluate the following integral in terms of gamma functions and compute the value.

$$\int_0^2 u^3(4-u^2)^{3/2}du$$ (8,10)

6. (a) Show that

$$u(x, t) = \sin(x)\ (vt)$$

is a solution to the one-dimensional wave equation

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 u}{\partial t^2}$$

(b) The temperature T(x, y) on each of the edges of a rectangular plate is maintained as

$$T(0, y) = 0,\ T(a, y) = 0,\ T(x, 0) = 0,\ T(x, b) = f(x).$$

Determine the steady state temperature distribution T(x, y) in the plate by solving Laplace equation

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0, \quad 0 < x < a,\ 0 < y < b$$

for $f(x) = T_0\, x\, (a - x)$, T_0 being a constant. (6,12)

(300)