$I_{n,m} = \frac{n-1}{m+n} I_{n-2,m}$, n and m are positive integers.

Also, evaluate $I_{4,5}$.

6. (a) Evaluate $\int_0^{\pi/2} \cos^n x \, dx$, where n is an odd positive integer.

(b) Trace the curve $r = a(1 + \cos \theta)$.

(c) Determine the intervals of concavity and points of inflexion of the curve

$y = 2x^4 - 3x^2 + 2x + 1$.

[This question paper contains 4 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 4605 **I**

Unique Paper Code : 2352571101

Name of the Paper : Topics in Calculus (DSC)

Name of the Course : **BA./B.Sc. (Prog.) with Mathematics as Non-Major/ Minor**

Semester : I

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll. No. on the top immediately on receipt of this question paper.

2. Attempt all question by selecting two parts from each question.

3. Parts of questions to be attempted together.

4. All questions carry equal marks.

1. (a) Show that $\lim_{x\to 0} \frac{e^{1/x}-1}{e^{1/x}+1}$ does not exist.

(b) If $y = e^{a\sin^{-1}x}$, prove that

$$(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+a^2)y_n = 0.$$

(c) State Euler's theorem and verify it for $f(x, y) = ax^2 + 2hxy + by^2$.

2. (a) Examine the continuity of the following function

$$f(x) = \begin{cases} x, & x \le 1 \\ 2-x, & 1 < x \le 2 \\ -2+3x-x^2, & x > 2 \end{cases}$$

at $x = 1$ and $x = 2$

(b) Find the n^{th} derivative of $y = \frac{x^2}{(x+2)(2x+3)}$.

(c) If $u = \log\frac{x^2+y^2}{x+y}$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 1$.

3. (a) Give the statement of Rolle's theorem and verify for $f(x) = \sqrt{1-x^2}$ in $[-1,1]$.

(b) Show that $\frac{x}{1+x} < \log(1+x) < x,\ x > 0$.

(c) Give the statement of Lagrange's mean value theorem and verify it for the function

$f(x) = x(x-1)(x-2)$ in $[0, \frac{1}{2}]$.

4. (a) Evaluate $\lim_{x\to\frac{\pi}{2}}(\sec x)^{\cot x}$.

(b) Give the statement of Taylor's theorem.

(c) Use Maclaurin's Series to expand $\sin x$.

5. (a) Find the asymptotes of the following curve

$xy^2 - x^2y - 3x^3 - 2xy + y^2 + x - 2y + 1 = 0$.

(b) Trace the curve $y(1-x^2) = x^2$.

(c) $I_{n,m} = \int_0^{\pi/2} \sin^n x \cos^m x\, dx$, prove that