5. (a) Find the asymptotes of the given curve

$$2x^3 - x^2y - 2xy^2 + y^3 - 4x^2 + 8xy - 4x + 1 = 0.$$

(b) Trace the curve $y^2 (a + x) = (a - x)x^2$.

(c) Evaluate using reduction formula

$$\int_0^{\infty} \frac{x^3}{(1+x^2)^{9/2}} dx.$$

6. (a) Obtain a reduction formula for $\int sin^m x \, cos^n x \, dx$, where m and n are positive integers. Hence, Evaluate $\int_0^{\pi/2} sin^3 x \, cos^4 x \, dx$.

(b) Trace the curve $r = 2 \sin 2\theta$.

(c) Determine the intervals of concavity and points of inflexion of the curve

$$y = x^4 - 4x^3 - 18x^2 + 1.$$

[This question paper contains 4 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 4793 **I**

Unique Paper Code : 2352571101

Name of the Paper : Topics in Calculus (DSC)

Name of the Course : **BA. / B.Sc. (Prog.) with Mathematics as Non-Major / Minor**

Semester : I

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll. No. on the top immediately on receipt of this question paper.

2. Attempt all question by selecting two parts from each question.

3. Parts of questions to be attempted together.

4. All questions carry equal marks.

1. (a) Show that $\lim_{x \to 0} \frac{e^{\frac{1}{x}} - e^{-\frac{1}{x}}}{e^{\frac{1}{x}} + e^{-\frac{1}{x}}}$ does not exist.

(b) If $y = a\cos(\log x) + b\sin(\log x)$, show that

$x^2 y_{n+2} + (2n + 1)xy_{n+1} + (n^2 + 1)y_n = 0.$

(c) State Euler's theorem and verify it for (x, y)

$= x^n \log\frac{y}{x}$.

2. (a) Show that the function

$$f(x) = \begin{cases} x, & 0 \le x < \frac{1}{2} \\ 1, & x = \frac{1}{2} \\ 1 - x, & \frac{1}{2} < x < 1, \end{cases}$$

is discontinuous at $x = \frac{1}{2}$. Also mention the type of discontinuity.

(b) Find the n^{th} derivative of $y = \frac{x^3}{x^2 - 3x + 2}$.

(c) If $u = \tan^{-1}\frac{x^3 + y^3}{x - y}$, $x \neq y$, prove that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u.$$

3. (a) Prove that $x < \sin^{-1} x < \frac{x}{\sqrt{1 - x^2}}$ if $0 < x < 1$.

(b) Give the statement of Lagrange's Mean Value theorem and verify for the function $f(x) = \log x$ in [1,e].

(c) Use Maclaurins's series to expand $\log(1 + x)$, for $-1 < x \le 1$.

4. (a) Evaluate $\lim_{x \to 0}\left(\frac{1}{x} - \cot x\right)$.

(b) Show that $\frac{x^2}{2(1+x)} < x - \log(1 + x) < \frac{x^2}{2}, x > 0$.

(c) Give the statement of Taylor's theorem.