This question paper contains 3 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **4595**

Unique Paper Code : **2352202002**

Name of the Course : **Theory of Equations and Symmetries**

Name of the Paper : **B.A. (Prog.)**

Type of the Paper : **DSC-3 (A or B)**

Semester : **III**

Duration : **3 Hours** Maximum Marks : **90**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *all* questions by selecting *two* parts from each question.

All questions carry equal marks.

Use of calculator is not allowed.

1. (*a*) Use synthetic division method to solve the polynomial equation $x^3 + x^2 + x + 1 = 0$.

   (*b*) Solve the cubic equation $2x^3 - 9x^2 + 12x - 4 = 0$, given that two of its roots are equal.

   (*c*) Determine the possible number of real solutions of the equation :

   $$4x^7 + 3x^6 + x^5 + 2x^4 - x^3 + 9x^2 + x + 1 = 0,$$

   using Descartes's rule of signs.

2. (*a*) If the difference between roots of the equation $x^2 - 13x + k = 0$ is 17. Find the value of k.

   (*b*) Solve the equation $2x^3 - x^2 - 22x - 24 = 0$, two of its roots being in the ratio 3 : 4.

   (*c*) Find all the values of $(\sqrt{3} + i)^4$.

P.T.O.

3. (*a*) Solve the following cubic equation by Cardon's method :

$$2x^3 - 7x^2 + 8x - 3 = 0.$$

(*b*) Find the equation whose roots are diminished by 3, the roots of :

$$x^4 - 7x^3 + 3x^2 - 11x + 17 = 0.$$

(*c*) Solve the following biquadratic equation by Descartes method :

$$x^4 - 8x^2 - 24x + 7 = 0.$$

4. (*a*) Find the solution of equation $x^3 - 18x + 35 = 0$ by Cardon's method.

(*b*) Find the equation whose roots are the roots of the equation $4x^5 - 2x^3 + 7x - 3 = 0$ each increased by 2.

(*c*) Find the solution of equation $x^4 - 3x^3 + x^2 + 3x - 2 = 0$ by Descartes method.

5. (*a*) Calculate the value of $\sum \alpha^3 \beta^3$ of the roots α, β and γ of the equation :

$$x^3 + px^2 + qx + r = 0.$$

(*b*) Find the value of the symmetric function $\sum \alpha^2 \beta^2$ of the roots of the biquadratic equation :

$$x^4 + px^3 + qx^2 + rx + s = 0.$$

(*c*) If α, β and γ are the roots of the cubic equation $x^3 + qx + r = 0$, then calculate the symmetric function $\sum \frac{\beta^2 + \gamma^2}{\beta + \gamma}$.

6. (*a*) Find the value of the symmetric function $\sum \frac{1}{\beta\gamma}$ for the roots of the equation :

$$x^3 + 3x^2 - 8x + 5 = 0.$$

(*b*) Find an equation whose roots are the translates of the roots of the equation :

$$x^4 - x^3 - 10x^2 + 4x + 24 = 0 \text{ by } 2.$$

(*c*) Use Newton's formula to show that for an equation of degree greater than equal to 4,

$$s_4 = c_1^4 - 4c_1^2c_2 + 4c_1c_3 + 2c_2^2 - 4c_4,$$

where, $s_k = \sum \alpha_1^k$, α_i's are the roots of the equation.