This question paper contains 3 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **4783**

Unique Paper Code : **2352202002**

Name of the Course : **Theory of Equations and Symmetries**

Name of the Paper : **B.A. (Prog.)**

Type of the Paper : **DSC-3 (A or B)**

Semester : **III**

Duration : **3 Hours** Maximum Marks : **90**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *all* questions by selecting *two* parts from each question.

All questions carry equal marks.

Use of calculator is not allowed.

1. (*a*) Find the quotient of $2x^4 - x^3 - 6x^2 + 4x - 8$ when divided by $x^2 - 4$.

   (*b*) If two roots of the equation $x^4 - 2x^3 - 12x^2 + 10x + 3 = 0$ are 1 and –3, then find the remaining two roots.

   (*c*) Find the nature of roots of the equation $x^4 - 3x^2 - x + 1 = 0$, using Descartes's Rule of Signs.

2. (*a*) If α and β are the roots of the equation $3x^2 + 7x - 2 = 0$, find the value of $\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}$.

   (*b*) Show that the *n*th roots of unity form a geometric progression.

   (*c*) Solve the equation $z^6 + z^3 + 1 = 0$.

P.T.O.

3. (*a*) Solve the following cubic equation by Cardon's method :

$$x^3 + 6x^2 + 3x + 18 = 0.$$

(*b*) Change the equation $3x^4 - 4x^3 + 4x^2 - 2x + 1 = 0$ into another, the coefficient of whose highest term will be unity.

(*c*) Solve the following biquadratic equation by Descartes method :

$$z^4 - 6z^2 - 16z - 15 = 0.$$

4. (*a*) Find the solution of equation $x^3 - 2x + 4 = 0$ by Cardon's method.

(*b*) Find the equation whose roots are the reciprocals of the roots of the equation :

$$x^6 + \frac{3}{4}x^5 - \frac{12}{5}x^4 + \frac{12}{5}x^2 - \frac{3}{4}x - 1 = 0.$$

(*c*) Find the solution of equation $x^4 - 8x^3 + 22x^2 - 24x + 9 = 0$ by Descartes method.

5. (*a*) Calculate the value of $\sum \alpha^3 \beta^3$ of the roote α, β and γ of the equation :

$$x^3 + px^2 + qx + r = 0.$$

(*b*) Find the value of the symmetric function $\sum \alpha^2 \beta^2$ of the roots of the biquadratic equation :

$$x^4 + px^3 + qx^2 + rx + s = 0.$$

(*c*) If α, β and γ are the roots of the cubic equation $x^3 + qx + r = 0$, then calculate the symmetric function $\sum \frac{\beta^2 + \gamma^2}{\beta + \gamma}$.

6. (*a*) Find the value of the symmetric function $\Sigma \frac{1}{\beta\gamma}$ for the roots of the equation :

$$x^3 + 3x^2 - 8x + 5 = 0.$$

(*b*) Find an equation whose roots are the translates of the roots of the equation :

$$x^4 - x^3 - 10x^2 + 4x + 24 = 0 \text{ by } 2.$$

(*c*) Use Newton's formula to show that for an equation of degree greater than equal to 4,

$$s_4 = c_1^4 - 4c_1^2c_2 + 4c_1c_3 + 2c_2^2 - 4c_4,$$

where, $s_k = \Sigma \alpha_1^k, \alpha_i' s$ are the roots of the equation.