(c) Give an example of a field with nine elements.

6. (a) Define an ideal of a Ring R. Prove that intersection of two ideal of a ring is an ideal. What about their union. Justify your answer.

(b) Let $S=\left\{\begin{bmatrix} a & b \\ -b & a \end{bmatrix} a, b \in R\right\}$. Show that $\varphi : \mathbb{C} \to S$ given by $\varphi(a+ib)=\begin{bmatrix} a & b \\ -b & a \end{bmatrix}$ is a Ring homomorphism.

(c) Let $\varphi : R \to S$ be a Ring homomorphism. Show that if R has unity 1 and $S \neq \{0\}$ and φ is onto then $\varphi(1)$ is the unity of S.

(700)

[This question paper contains 4 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 5415 **H**

Unique Paper Code : 2352572401

Name of the Paper : Abstract Algebra

Name of the Course : **B.A. / B.Sc. (Prog.) – DSC**

Semester : IV

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **all** question by selecting two parts from each question.
3. Part of the questions to be attempted together.
4. **All** questions carry equal marks.
5. Use of Calculator not allowed.

P.T.O.

1. (a) Describe each symmetry in D_3 (the set of symmetries of an equilateral triangle). Prove that D_3 is a group.

   (b) (i) Give two reasons why the set of odd integers under addition is not a group.

   (ii) If a and b are elements of a group G and $ab \neq ba$, then prove that $aba \neq e$, where e is the identity of G.

   (c) Define an Abelian group. If G is a group with the property that every element is its own inverse then prove that G is Abelian.

2. (a) Define the order of an element in a group G. Find the order of each element in the group U(12).

   (b) Prove that the centre of a group G is a subgroup of G.

   (c) Suppose that H is a proper subgroup of the set Z of integers under addition and H contains 18, 30, and 40. Determine H.

3. (a) Let $G = \langle a \rangle$ be a cyclic group of order n. Then show that $G = \langle a^k \rangle$ if and only if $\gcd(k, n) = 1$.

   (b) Let

$$\alpha = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 2 & 1 & 3 & 5 & 4 & 7 & 6 & 8 \end{bmatrix} \quad \text{and}$$

$$\beta = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 1 & 3 & 8 & 7 & 6 & 5 & 2 & 4 \end{bmatrix}$$

   Compute the following :

   (i) Write α and β as product of disjoint cycles.

   (ii) Compute αβ and α^{-1}.

   (c) Suppose that a has order 15. Compute all the left cosets of $\langle a^5 \rangle$ in $\langle a \rangle$.

4. (a) State Lagrange's theorem for groups. Show that group of prime order is cyclic.

   (b) Write all the elements of the group A_4. If H = {(1), (1 2)(3 4)}, show that H is a subgroup of A_4 but not normal subgroup of A_4.

   (c) Define group homomorphism. Determine all homomorphism from Z_n.

5. (a) Describe all the subring of the ring of integers.

   (b) Define an Integral Domain. Show that a finite Integral Domain is a field.