5. (a) Find the equation of the sphere with centre at (1,2,3) and touching a plane at (2,1,3).

(b) Prove that the equation $ax^2 + by^2 + cz^2 + 2ux + 2vy + 2wz + d = 0$ represents a cone if

$$\frac{u^2}{a}+\frac{v^2}{b}+\frac{w^2}{c}=d.$$

(c) Find equation of cylinder whose guiding curve is $x^2 + y^2 = 16$, $z = 0$ and generators are parallel to the line $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$.

6. (a) Find the equation of the sphere with centre (0,1,0) and touching the plane $x - 2y + 2z + 5 = 0$.

(b) Find the equation of the right circular cone whose vertex is at the origin and whose axis is the line $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ and whose semi vertical angle is 30°.

(c) Show that the line $\frac{x}{1}=\frac{y}{-1}=\frac{z-3}{1}$ is a generator of the cylinder $x^2 + y^2 + z^2 + xy + yz - zx = 9$.

[This question paper contains 4 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 5487 **H**

Unique Paper Code : 2352201202

Name of the Paper : DSC : Analytic Geometry

Name of the Course : **B.A. (Programme) with Mathematics as Major**

Semester : II

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** questions are compulsory.
3. Attempt **two** parts from each question.
4. Each question carries equal marks.
5. Use of Calculator not allowed.

1. (a) Find an equation for the ellipse that satisfies the given conditions : foci (–1, 1) and (–1, 3); minor axis of length 4.

   (b) Sketch the graph of the hyperbola :

   $$x^2 - y^2 - 4x + 8y - 21 = 0.$$

   (c) Find the equation of the parabola that has its vertex at (5, –3), axis parallel to y-axis and passes through (9, 5).

2. (a) Identify and sketch the curve $xy = -9$.

   (b) Sketch the following parabola and label its focus, vertex and directrix :

   $$y^2 - 2x - 6y + 1 = 0.$$

   (c) Sketch the graph of the following ellipse and label its foci, vertices and ends of minor axis :

   $$9x^2 + 4(y + 2)^2 = 36.$$

3. (a) Express the vector v as the sum of a vector parallel to b and a vector orthogonal to b, where $v = 4i - 2j + 6k$ and $b = -2i + j - 3k$.

   (b) Find the volume of the parallelepiped that has $u = 3i + j + 2k$, $v = 4i + 5j + k$, and $w = i + 2j + 4k$ as the adjacent edges. Also find two unit vectors that are orthogonal to the plane determined by v and w.

   (c) Find the parametric equations for the line that is perpendicular to the lines L_1 and L_2 and passes through their point of intersection.

   L_1 : $x = 4t$, $y = 1 - 2t$, $z = 2 + 2t$

   L_2 : $x = 1 + t$, $y = 1 - t$, $z = -1 + 4t$.

4. (a) Find the equation of the plane that passes through the point (–1,4,2) and contains the line of intersection of the planes

   $4x - y + z - 2 = 0$ and $2x - y - 2z - 3 = 0$.

   (b) Determine whether the lines L_1 and L_2 are parallel, intersect, or skew.

   L_1 : $x = -1 + 4t$, $y = 3 + t$, $z = 1$

   L_2 : $x = -13 + 12t$, $y = 1 + 6t$, $z = 2 + 3t$.

   (c) In each part, find two unit vectors in 3-space that satisfy the stated conditions.

   (i) Perpendicular to the xy-plane

   (ii) Perpendicular to the xz-plane

   (iii) Perpendicular to the yz-plane