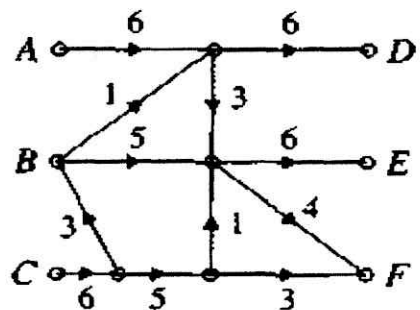




[This question paper contains 12 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 5497

H

Unique Paper Code : 2352202402

Name of the Paper : Introduction to Graph Theory

Name of the Course : **B.A. (Programme) with
Mathematics as Major
(DSC-2)**

Semester : IV

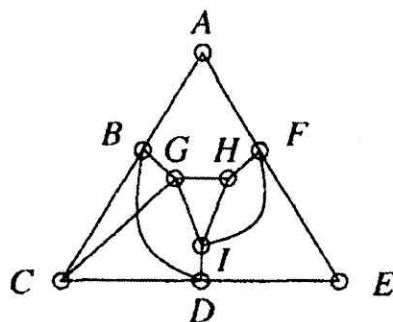
Duration : 3 Hours

Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **all** questions by selecting **two** parts from each question.
3. Both parts of a question to be attempted together.
4. **All** questions carry equal marks.

1. (a) Write the Adjacency Matrix of a complete bipartite graph on the sets $V_1 = \{a, b\}$ and $V_2 = \{c, d, e\}$. Why the diagonal entries of Adjacency matrix of a graph have value zero? Is Adjacency matrix symmetric? (7.5)
- (b) Define a complete bipartite graph. Write degree sequence in the graph $K_{4,5}$. How many edges does $K_{2,6}$ contain. (7.5)
- (c) Define Hamiltonian graph. Is the following graph Hamiltonian? Justify. (7.5)



communities in the far north. The warehouses have, respectively, 500, 500, and 900 snow blowers in stock on October 1. The first blizzard of winter is forecast to start near midnight, October 2, and there is an immediate demand from the stores for 700, 600, and 600 snow blowers, respectively. Various routes are available for shipping merchandise to the stores. These are shown in the following Fig., where the capacity of an arc uv is the largest number of snow blowers that can be shipped from u to v in the course of a single day. The vertices in the middle should be thought of as middlemen, baggage handlers, for example, who can only cope with so many snow blowers at once. We will assume that freight can cover as many arcs as necessary in a single day. Can all the required snow blowers reach the stores before the blizzard arrives? If not, how close can the company come to satisfying demand? (7.5)

- (ii) Find the capacity of the (s,t) -cut defined by $S = \{s, a, b, c, d, e\}$ and $T = \{t\}$.

(7.5)

- (b) Define Matching in Graph. Five ladies have men friends as shown in the following table

Karen	David, Stuart, Paul, Roger
Mary	Stuart, Philip
Aurie	David, Paul
Pamela	Paul, Roger
Lynne	Stuart, Paul

Draw a bipartite graph depicting this situation. Find a way in which each lady can marry a man she knows, or use Hall's Marriage Theorem to explain why no such matching exists.

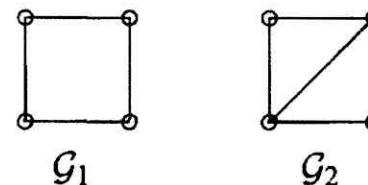
(7.5)

- (c) A small department store chain maintains three warehouses, A, B, and C, in the south of the province and three stores, D, E, and F, in rural

2. (a) When are two graphs said to be isomorphic? Suppose that two graphs have the same number of edges and suppose that degree of every vertex in both the graphs is 2. Are two graphs necessarily isomorphic?

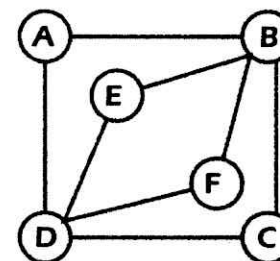
Are the following two graphs isomorphic. Justify.

(7.5)



- (b) Define an Eulerian circuit. Is the following graph Eulerian. Justify. Write an Eulerian circuit in it.

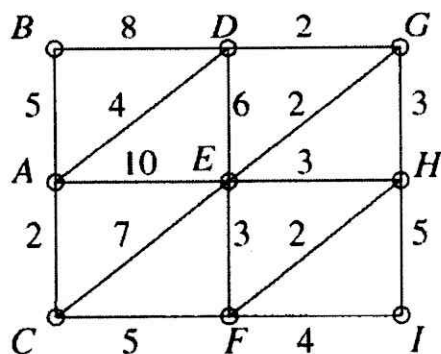
(7.5)



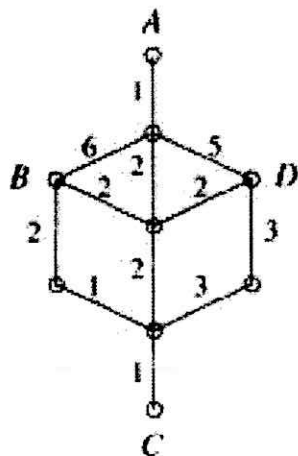
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- (c) Use DIJKSTRA'S Algorithm to find shortest path from vertex A to all other vertices in a weighted graph. (7.5)



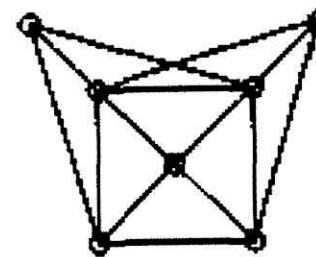
3. (a) Solve the Chinese postman problem for the following weighted graph : (7.5)



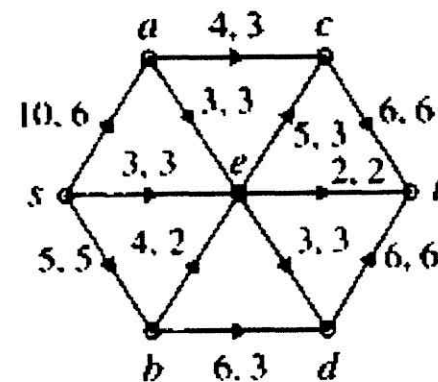
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- (c) Define coloring of graph. What is Chromatic Number? Find Chromatic Number of the graph. (7.5)

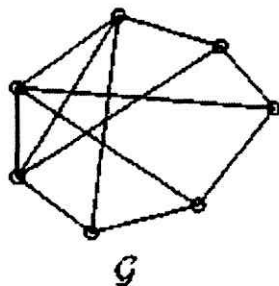


6. (a) For the network shown.



- (i) Verify the law of conservation of flow at a, e, and d.

P.T.O.



(b) Suppose that in one particular semester there are students taking each of the following combinations of courses.

1. Mathematics, Physics, French.
2. Mathematics, English, German.
3. English, French, Physics.
4. Chemistry, Physics, French.

What is the minimum number of examination period required for exams in the specified courses so that students involved have no conflicts? Find a possible schedule that uses this minimum number of periods. (7.5)

(b) To finish a basement, a contractor must arrange for certain task to be completed.

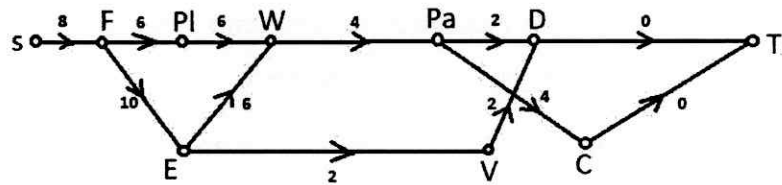
As shown in below table, brief codes for reference and the time required for each activity. (7.5)

Task	Code	Time
Floor installation	F	8
Plumbing	P	6
Electrical work	E	10
Wallboard	W	6
Varnish doors and moldings	V	2
Paint	Pa	4
Install doors and moldings	D	2
Lay Carpet	C	4

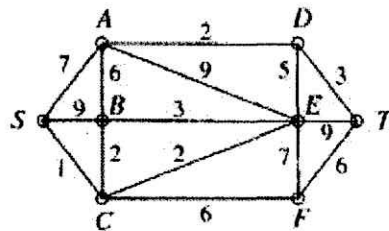
Referring to the above graph, find the minimum time to complete the task.

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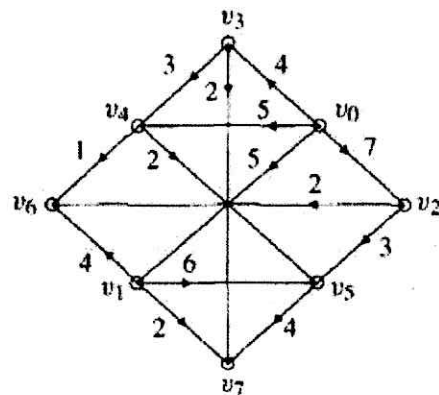
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- (c) Find the minimum spanning tree for the following graph (starting with the vertex S). (7.5)



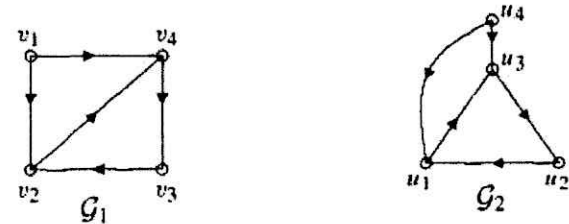
4. (a) Apply Bellman's algorithm to find the lengths of shortest paths from v_0 to each of other vertices. (7.5)



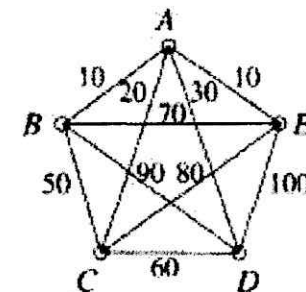
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- (b) Show that the following graphs are isomorphic. (7.5)



- (c) Solve the Travelling Salesman's Problem for the following graph. (Start at A) (7.5)



5. (a) State Kuratowski Theorem. Show that given graph is not Planar. (7.5)

P.T.O.