(c) Suppose that φ is a homomorphism from U(30) to U(30) and that Ker $\varphi = \{1,11\}$. If $\varphi(7) = 7$, find all elements of U(30) that map to 7.

5. (a) Show that the set $\left\{\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \middle| a, b \in Z\right\}$ of diagonal matrices is a subring of the ring of all 2×2 matrices over Z.

(b) Prove that Z_n is a field if and only if n is prime.

(c) Define an Integral domain. Show that the ring $M_2(Z)$ of 2×2 matrices over integers is not an Integral domain.

6. (a) Define factor ring. Write all the elements of $\frac{z}{4z}$ and prove that it is a ring.

(b) Let $S = \{a + ib \mid a, b \in Z \text{ and } b \text{ is even}\}$. Show that S is a subring of Z[i] but not an ideal of Z[i].

(c) Let $\phi : R \to S$ be a ring homomorphism. If A is an ideal and ϕ is onto then show that the $\phi(A)$ is an ideal of S.

[This question paper contains 4 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 5596 **H**

Unique Paper Code	:	2352572401
Name of the Paper	:	Abstract Algebra
Name of the Course	:	**B.A. / B.Sc. (Prog.) – DSC**
Semester	:	IV

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **all** question by selecting two parts from each question.
3. Part of the questions to be attempted together.
4. **All** questions carry equal marks.
5. Use of Calculator not allowed.

1. (a) If $G = \left\{ \begin{bmatrix} a & a & a \\ a & a & a \\ a & a & a \end{bmatrix}, a \in R, a \neq 0 \right\}$. Prove that G is a group under matrix multiplication. Explain why each element of G has an inverse even though the matrices have 0 determinants.

   (b) Show that the set A = {1,2,3} under multiplication modulo 4 is not a group but that B = {1,2,3,4} under multiplication modulo 5 is a group.

   (c) For any integer $n > 2$, prove that there are at least two elements in U(n) that satisfy $x^2 = 1$.

2. (a) Let G be a group and H a nonempty subset of G. Prove that H is a subgroup of G if and only if ab^{-1} is in H whenever a and b are in H.

   (b) Define centralizer of an element a of a group G and if order of a is 5. Prove that $C(a) = C(a^3)$.

   (c) Let H and K be two subgroups of G, show that $H \cap K$ is a subgroup of G. Is it also true for $H \cup K$? Justify.

3. (a) Show that U(10) with the operation of multiplication modulo 10 is cyclic. Find all its generators.

   (b) Describe symmetries of an equilateral triangle.

   (c) Define an even and an odd permutation. Determine whether the following permutations are even or odd :

      (i) $\sigma = (1\ 3\ 5\ 6\ 7)$

      (ii) $\gamma = (1\ 2)(1\ 3\ 4)(1\ 5\ 2)$

      (iii) $\beta = (1\ 2\ 4\ 3)(3\ 5\ 2\ 1)$

4. (a) Let H be a subgroup of G and a, b be elements of G. prove that

      (i) $aH = H$ if and only if $a \in H$

      (ii) $o(aH) = o(bH)$.

   (b) Define a normal subgroup. Let

$$H = \left\{ \begin{bmatrix} a & b \\ 0 & d \end{bmatrix} : a, b, d \in \mathbb{R}, ad \neq 0 \right\}.$$

   Is H a normal subgroup of $GL(2, \mathbb{R})$.