5. (a) Let a be an element of a ring R and consider the set $S = \{x \in R \mid ax = 0\}$. Show that S is a subring of R.

(b) Define an integral domain and field. Is every field an integral domain? What about converse. Justify your answer.

(c) Show that characteristics of an integral domain is zero or prime.

6. (a) Let R be a commutative ring and $a \in R$, show that $aR = \{ar \mid r \in R\}$ is an ideal of R.

(b) Let A be an ideal of a ring R. Prove that the mapping $r \rightarrow r + A$ is a ring homomorphism from R to R/A. Also prove that its kernel is A.

(c) Define factor ring. Write all the elements of $\frac{2Z}{6Z}$ and prove that it is a ring.

[This question paper contains 4 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 5470 **H**

Unique Paper Code : 2352572401

Name of the Paper : Abstract Algebra

Name of the Course : **B.A. / B.Sc. (Prog.) – DSC**

Semester : IV

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. Attempt **all** question by selecting two parts from each question.

3. Part of the questions to be attempted together.

4. **All** questions carry equal marks.

5. Use of Calculator not allowed.

1. (a) Describe each symmetry in D_4 (the set of symmetries of an equilateral triangle). Prove that D_4 is a group.

   (b) (i) Prove that a group G is Abelian if and only if $(ab)^{-1} = a^{-1}b^{-1}$ for all a and b in G.

   (ii) Give two reasons why the set of odd integers under addition is not a group.

   (c) Prove that the set of all rational numbers of the form 3^m6^n, where m and n are integers, is a group under multiplication.

2. (a) If G be an Abelian group and H and K be subgroups of G. Then prove that HK = {hk: h $\in$ H, k $\in$ K) is a subgroup of G.

   (b) If H and K are subgroups of G, show that $H \cap K$ is a subgroup of G. Is it also true for $H \cup K$? Justify.

   (c) Suppose n is an even positive integer and H is a subgroup of Z_n every member of H is even or exactly half of the members of H are even.

3. (a) Define a cyclic group. Prove that every cyclic group is Abelian. Is the converse true? Justify.

   (b) Let a be an element of a finite group G with identity e. Prove that $a^{o(G)} = e$. Prove that a group of order 3 must be cyclic.

   (c) What is the order of each of the following :

   (i) (1 2 3)(4 5 6)

   (ii) (1 2 4)(3 5 7 8 6 9)

   (iii) (2 4 3 5)(1 2 5 4 6 7)

4. (a) Prove that the center Z(G) of a group G is a normal subgroup of G.

   (b) Consider the subgroup H = {1,9} of group G = U(20) under multiplication modulo 20. Find the number of cosets of H in G and determine all the distinct cosets of H in G.

   (c) Let G be a group of permutations and consider the multiplicative group $\{1, -1\}$. For each $\sigma \in G$, define $\Phi : G \to \{1, -1\}$ by

$$\Phi(\sigma) = \begin{cases} 1, & \sigma \text{ is an even permutation} \\ -1, & \sigma \text{ is an odd permutation} \end{cases}$$

   Prove that Φ is a homomorphism. Also, find Ker Φ.