(b) Find the matrix $A_{BC}$ for linear transformation $L: \mathbb{R}^4 \rightarrow \mathbb{R}^2$ defined as

$$L([x, y, z, w]) = [3x - 5y + z - 2w, 5x + y - 2z + 8w]$$

with respect to the standard basis $B = (e_1, e_2, e_3, e_4)$ for $\mathbb{R}^4$ and standard basis $C = (e_1, e_2)$ for $\mathbb{R}^2$.

(c) Let $L: V \rightarrow W$ be a linear transformation. Prove that kernel of L is a subspace of V and range of L is a subspace of W.

6. (a) Let $L: P_2 \rightarrow P_3$ be a linear transformation given by

$$L(a\,x^2 + b\,x + c) = cx^3 + bx^2 + ax.$$

Find the basis for kernel of L and basis for range of L.

(b) Let $L: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be a linear transformation defined as

$$L\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}.$$

Prove that L is one-to-one and onto.

(c) Determine whether the linear transformation $L: P_3(x) \rightarrow P_2(x)$ defined as $L(p(x)) = p'(x)$ where $p'(x)$ is the derivative of $p(x)$ and $P_n(x)$ is the vector space of all polynomials of degree atmost n is an isomorphism.



[This question paper contains 4 printed pages.]

**Your Roll No..............**

**Sr. No. of Question Paper : 5587** **H**

Unique Paper Code : 2352571201

Name of the Paper : Elementary Linear Algebra

Name of the Course : **B.A. / B.Sc. (Prog.) with Mathematics as Non-Major/ Minor – DSC**

Semester : II

Duration : 3 Hours Maximum Marks : 90

**Instructions for Candidates**

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** questions are compulsory.
3. Questions **1** to **6** have three parts each. Attempt any **TWO** parts from each Question. Each part carries **7.5** marks.
4. Use of Calculator not allowed.

1. (a) Which of the following matrices are in row echelon form. If not, then state the reason

$$\begin{bmatrix} 1 & 2 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

(b) Define rank of a matrix. Find the rank of the matrix

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 3 & 2 & 1 \end{bmatrix}.$$

(c) Using Gaussian Elimination Method, Find the cubic equation $y = ax^3 + bx^2 + cx + d$ that goes through the points (1,1), (2, −18), (−2, 46) and (3, −69).

2. (a) State Triangle Inequality. Verify it for the vectors [−1, 3, 4, 1] and [3, 0, 2, −1] in $\mathbb{R}^4$.

(b) Let $x = [14, -21, 7] \in \mathbb{R}^3$ and $S = \{[2, -3, 1], [-4, 6, -2]\}$ be a subset of $\mathbb{R}^3$. Is it possible to express x as a linear combination of elements of set S? If yes, then find the expression.

(c) Solve the following system of equations using Gaussian Elimination method, if consistent :

$$\begin{aligned} 2x_1 + 3x_2 - x_3 &= 9, \\ x_1 + x_2 + x_3 &= 9, \\ 3x_1 - x_2 - x_3 &= -1, \\ -4x_1 - 6x_2 + 2x_3 &= -18. \end{aligned}$$

3. (a) Find the eigen value of the matrix and the corresponding eigen space

$$A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}.$$



(b) Prove that the subset $W = \{[a, b, \frac{1}{2}a - 2b]\}$ is a subspace of $\mathbb{R}^3$ under the usual operations of vector addition and scalar multiplication.

(c) Let S = {[1, 3, −1], [2,7,3], [4, 8, −7]} be a subset of $\mathbb{R}^3$. Using Simplified Span method, prove that S spans $R^3$.

4. (a) Prove or disprove that the following matrix is diagonalizable

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}.$$

(b) Determine whether the subset S = {[2, −1, 0, 5], [1, −1, 2, 0], [−1, 0, 1, 1]} of $\mathbb{R}^4$ is linearly independent or not.

(c) Define a basis of a vector space V. Find the basis of the subspace

$W = \{[x,y,z] \in \mathbb{R}^3 : x + 5y + z = 0\}$ of $\mathbb{R}^3$.

5. (a) Prove that the function $g: M_{nn} \to M_{nn}$ defined as $g(A) = A^t$, where $A^t$ denotes the transpose of the matrix and $M_{nn}$ is the vector space of all $n \times n$ matrices is a linear operator.