

7. (a) If $\{X_n\}$ be a sequence of random variables and

$$Y_n = \frac{S_n - E(S_n)}{n}; S_n = X_1 + X_2 + \dots + X_n,$$

then $\lim_{n \rightarrow \infty} E\left(\frac{Y_n^2}{1+Y_n^2}\right) \rightarrow 0$ is the necessary and sufficient condition for WLLN to hold good.

- (b) Let $X_1, X_2, \dots, X_n$ are independent observations from

$$N(0,1) \text{ and } \Phi(z) = \int_{-\infty}^z \frac{1}{\sqrt{2\pi}} e^{-\frac{\theta^2}{2}} d\theta. \text{ Show that}$$

$$E(\text{Min}(|X_1|, |X_2|, \dots, |X_n|)) = 2^n \int_0^\infty (1 - \Phi(z))^n dz.$$

8. (a) Discuss the test of significance for the difference of means $(\mu_1 - \mu_2)$ of two different populations when large samples from each population are available. Also obtain $100(1 - \alpha)\%$ confidence interval for the difference of means.

- (b) Explain the term standard error of sampling distribution. Show that in a sequence of n independent Bernoulli trials with constant probability θ of success, the standard error of

$$\text{proportional of successes is } \sqrt{\frac{\theta(1-\theta)}{n}}.$$

(1000)

[This question paper contains 4 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 4068

H

Unique Paper Code : 2372012401

Name of the Paper : Sampling Distributions

Name of the Course : **B.Sc. (Hons) Statistics**
(NEP-UGCF)

Semester : IV

Duration : 3 Hours

Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **Six** questions in all by selecting **three** questions from each section.
3. All questions/parts carry equal marks.

Section – A

1. (a) Derive Brandt and Snedecor formula for computing Chi-square test statistics from observations available in the form of contingency table of size $2 \times k$.

P.T.O.

- (b) Let $X_1, X_2, \dots, X_n$ be a random sample from $N(\mu, \sigma^2)$ with sample mean $(\bar{X})$ and sample

variance $S^2 = \frac{1}{n-1} \sum_{i=1}^n (X_i - \bar{X})^2$. Let X_{n+1} be

another observation from the same population and independent of $X_1, X_2, \dots, X_n$. Obtain sampling

distribution of $W = \frac{X_{n+1} - \bar{X}}{S} \sqrt{\frac{n}{n+1}}$.

2. (a) For a Chi-Square distribution with n degree of freedom, establish the following recurrence relation between the moments: $\mu_{r+1} = 2r (\mu_r + n \mu_{r-1})$, $r \geq 1$. Hence, find μ_r ; $r = 1, 2, 3, 4$.

- (b) Let $X \sim \chi_{(n)}^2$ and if $p_x = P(X \leq x)$ then, show that

$$x < \frac{n}{1 - p_x}.$$

3. (a) For t - distribution with $n = 4$ degree of freedom,

show that $P(t \geq 2) = \frac{1}{2} - \frac{5\sqrt{2}}{16}$.

- (b) Let X follows t - distribution with n degree of freedom, then, show that $Y = \frac{n}{n+X^2}$ follows Beta distribution.

4. (a) Define F-statistics and derive formula for its probability density function.

- (b) If $X \sim F(m, n)$ then, show that

$$Y = \frac{(m/n)X}{1 + (m/n)X} \sim B\left(\frac{m}{2}, \frac{n}{2}\right).$$

Deduce variance of X from the variance of Y .

Section - B

5. (a) Let $X_1, X_2, \dots, X_n$ be a n i.i.d Poisson variates with parameter λ . Use CLT and function

$$\Phi(x) = (2\pi)^{-\frac{1}{2}} \int_{-\infty}^x e^{-\frac{\theta^2}{2}} d\theta \text{ to estimate } P(120 \leq S_n \leq 160), \text{ where } S_n = X_1 + X_2 + \dots + X_n; \lambda = 2 \text{ and } n = 75.$$

(b) Show $\lim_{n \rightarrow \infty} \left(\int_0^n \frac{e^{-t} t^{n-1}}{(n-1)!} dt \right) = \frac{1}{2}$.

6. (a) Let $X_1, X_2, \dots, X_n$ be n independent observations from exponential distribution with mean μ and having Range $= X_{(n)} - X_{(1)}$. Then, show that

$$E(\text{Range}) = \left(1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n-1} \right) \mu.$$

- (b) Prove that convergence in mean square implies convergence in probability.