5. (a) Evaluate $\oint_C \left(x^2 y\,dx - xy\,dy\right)$, where C is the path that begins at (0,0), goes to (1,1) along the parabola $y = x^2$, and then return to (0,0) along the line $y = x$.

(b) Prove or disprove that the line integrals are path independent.

(c) Evaluate the line integral

$$\oint_C [(2x - x^2y)e^{-xy} + \tan^{-1} y]\boldsymbol{i} + [\tfrac{x}{y^2+1} - x^3e^{-xy}]\boldsymbol{j}.\,dR,$$

where C, the curve with parametric equations $x = t^2 \cos \pi t$, $y = e^{-t} \sin \pi t$, $0 \le t \le 1$.

6. (a) Find the area enclosed by the semicircle $y = \sqrt{4 - x^2}$ using the line integral.

(b) Find the mass of the homogenous lamina of density $\delta = x$ that has the shape of the surface S given by $z = 4 - x - 2y$ with $z \ge 0$, $x \ge 0$ and $y \ge 0$.

(c) Let $F = 2xi - 3yj + 5zk$, and let S be the hemisphere $z = \sqrt{9 - x^2 - y^2}$ together with the disk $x^2 + y^2 \le 9$ in the xy–plane. Verify the divergence theorem.

[This question paper contains 4 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 4102 **H**

Unique Paper Code : 2352012402

Name of the Paper : Multivariate Calculus

Name of the Course : **B.A./B.Sc. (H)**

Semester : IV (DSC)

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **all** questions by selecting **two** parts from each question.
3. **All** questions carry equal marks.
4. Use of Calculator not allowed.

1. (a) Let f be the function defined by

$$f(x,y) = \begin{cases} \dfrac{xy^3}{x^2 + y^6}, & (x,y) \ne (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$$

Is f continuous at (0,0)? Explain.

(b) A cylindrical tank is 4 ft high and has an outer diameter of 2 ft. The walls of tank are 0.2 inches thick. Approximate the volume of interior of tank assuming the tank has a top and a bottom that are both 0.2 inches thick.

(c) Find an equation for each horizontal tangent plane to the surface

$$z = 5 - x^2 - y^2 + 4y.$$

2. (a) Let $f(x, y, z) = ye^{x+z} + ze^{y-x}$. At the point (2,2,–2) find the unit vector pointing in the direction of most rapid increase of f. And what is the value of most rapid increase of f?

(b) Find the absolute extrema of the function $f(x, y) = 2 \sin x + 5 \cos y$ on the set S where S is the rectangular region with vertices (0,0), (2,0), (2,5) and (0,5).

(c) Find the point on the plane $2x + y + z = 1$ that is nearest to the origin.

3. (a) (i) Sketch the region of integration and compute the double integral

$$\int_0^{\pi/2} \int_0^{\sin x} e^x \cos x \, dydx .$$

(ii) Evaluate $\int\int_D (2y-x)dA$ where D is the region bounded by $y = x^2$, $y = 2x$.

(b) Evaluate the area bounded by $r = 2 \cos\theta$.

(c) Evaluate the given integral by converting to polar coordinates

$$I = \int_0^2 \int_y^{\sqrt{8-y^2}} \frac{1}{\sqrt{1+x^2+y^2}} dydx .$$

4. (a) (i) Set up a triple integral to find volume of solid bounded above by paraboloid $z = 6 - x^2 - y^2$ and below by $z = 2x^2 + y^2$.

(ii) Set up a double integral to find volume of solid region bounded by ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1 .$$

(b) Use cylindrical co-ordinates to compute integral $\iiint_D z\left(x^2+y^2\right)^{-1/2} dxdydz$ where D is the solid bounded above by plane $z = 2$ and bounded below by surface $2z = x^2 + y^2$.

(c) Find volume of solid D where D is the intersection of solid sphere $x^2 + y^2 + z^2 \leq 9$ and solid cylinder $x^2 + y^2 \leq 1$.