(ii) Show that 0.01010101 ... .. converges and find it's sum.

6. (a) State nth Root Test (Basic Form). Using this test or otherwise, test the convergence of the following series :

(i) $\sum_{n=1}^{\infty}\left(\frac{4n-3}{2n+1}\right)^{n}$

(ii) $\sum_{n=1}^{\infty}\left(\frac{n^2+2n}{n^4+1}\right)^{n}$

(b) Test the convergence of following series :

(i) $\sum_{n=2}^{\infty}\frac{n}{2^n}$

(ii) $\frac{1}{3.7}+\frac{1}{4.9}+\frac{1}{5.11}+\frac{1}{6.13}+\cdots$

(c) (i) State Alternating Series Test. Show that alternating harmonic series $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}$ converges.

(ii) Test the convergence and absolute convergence of the series $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n^2+1}$.

(200)

[This question paper contains 4 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 5649 **H**

Unique Paper Code : 2354002003

Name of the Paper : Elements of Real Analysis

Name of the Course : **COMMON PROG GROUP-GE**

Semester : IV

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **all** question by selecting two parts from each question.
3. Part of the questions to be attempted together.
4. **All** questions carry equal marks.
5. Use of Simple Calculator allowed.

1. (a) Let F be a complete ordered field, A and B are nonempty subsets of F bounded above in F. Prove that A ∪ B is bounded above, and sup(A ∪ B) = max {sup A, sup B}.

(b) Verify whether all the field axioms for the set $\mathbb{R}^2$ with $(x, y) + (u, v) = (x + u, y + v)$ and $(x, y).(u, v) = xu + yv$ are satisfied or not.

(c) Prove that for all $x, y \in$ ordered field F,

(i) $||x| - |y|| \leq |x + y|$.

(ii) $0 \leq x < y$ implies $\frac{x}{1+x} < \frac{y}{1+y}$.

2. (a) Let F be an Archimedean ordered field, $A \subseteq F$ and $u \in F$. Then prove that $u = \inf A$ if and only if for each $\in > 0$,

(i) For all $x \in A$, $x > u - \in$ and

(ii) There exist $x \in A$ such that $x < u + \in$.

(b) Show that the set $S=\{\frac{1}{n} : n \in \mathbb{N}\}$ is bounded above and below and give its supremum and infimum.

(c) Let $<x_n>$ be a sequence of real numbers defined inductively by $x_1 = 1$ and $x_{n+1} = \sqrt{6+x_n}$, for all $n \in N$. Prove that the sequence $<x_n>$ converges and find its limit.

3. (a) Use the definition of the limit of a sequence to find the following limit:

(i) $\lim\limits_{n\to\infty} \frac{3n+1}{2n+5}$ (ii) $\lim\limits_{n\to\infty} \frac{1}{\sqrt{n+2}}$

(b) Prove that if $c > 0$, then $\lim\limits_{n\to\infty} c^{\frac{1}{n}} = 1$.

(c) State and prove Monotone convergence theorem.

4. (a) Prove that every convergent sequence of real numbers is a Cauchy sequence. Is the sequence $<x_n>$, where $x_n = \frac{n+1}{n}$, a Cauchy sequence. Justify your answer.

(b) Prove that the sequence $<x_n>$, where $x_n = \left(1+\frac{1}{n}\right)^n$, is convergent.

(c) Show that, for any fixed real number c, $\lim\limits_{n\to\infty} \frac{c^n}{(n)!} = 0$.

5. (a) Show that the telescoping series $\sum_{n=1}^{\infty} \frac{1}{n^2+5n+6}$ converges and find it's sum.

(b) Determine whether the series $\sum_{n=1}^{\infty} ln\left(\frac{n+2}{n+1}\right)$ converges or diverges.

(c) (i) Determine whether the geometric series $\sum_{n=0}^{\infty} \frac{1}{15}\left(\frac{-5}{2}\right)^n$ converges or diverges. Find the sum of the series if it converges.