

6081

8

$$L([x,y,z]) = [-2x + 3z, x + 2y - z].$$

Find the matrix representation for L with respect to the basis

$B = \{[1,-3,2], [-4,13,-3], [2,-3,20]\}$ and $C = \{[-2,-1], [5,3]\}$ for $\mathbb{R}^3$ and $\mathbb{R}^2$ respectively.

(1000)

[This question paper contains 8 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 6081

H

Unique Paper Code : 2354001202

Name of the Paper : Introduction to Linear Algebra

Name of the Course : GE II

Students : Regular

Semester : II

Duration : 3 Hours

Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **all** question by selecting two parts from each question.
3. **All** questions carry equal marks.

P.T.O.

1. (a) Find the angle between the vectors $x = [8, -20, 4]$, $y = [6, -15, 3]$. Are vectors x and y parallel? Justify your answer. Also, verify the answer by finding $c \in \mathbb{R}$ such that $x = cy$.
- (b) Use the Gaussian elimination method to find the values of the scalars a , b and c which satisfy the quadratic equation $y = ax^2 + bx + c$ that passes through the points $(3,20)$, $(2,11)$ and $(-2,15)$ in the xy -plane.
- (c) Use the Gauss-Jordan method to convert the augmented matrix to reduced row echelon form:

$$\left[\begin{array}{ccc|c} 3 & 1 & -2 & 1 \\ 4 & 0 & -1 & 7 \\ 2 & -3 & 5 & 18 \end{array} \right]$$

2. (a) Let x and y be vectors in $\mathbb{R}^n$ then prove that $\|x + y\|^2 + \|x - y\|^2 = \|x\|^2 + \|y\|^2$ if and only if $x \cdot y = 0$.

$$L \left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \right) = \begin{bmatrix} 3 & 2 & 11 \\ 2 & 1 & 8 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

- (i) Is $[-5, 2, 1]$ in $\ker(L)$? Justify the answer.
- (ii) Is $[3, 2]$ in $\text{range}(L)$? Justify the answer.
- (b) Let $L: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ be a linear transformation given by:

$$L \left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \right) = \begin{bmatrix} -7 & 4 & -2 \\ 16 & -7 & 2 \\ 4 & -3 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Find the dimensions of the kernel of L and the range of L . Use them to determine that whether the linear transformation L is one-one or not, and onto or not.

- (c) Let $L: \mathbb{R}^3 \rightarrow \mathbb{R}^2$ be a linear transformation given by:

- (ii) $L_2: P_2 \rightarrow P_3$ given by $L_2(p(x)) = x^3p'(0) + x^2p'(0) + 1$, where $p'(x)$ denotes the derivative of $p(x)$ with respect to x .

- (b) Let $L: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ be a linear transformation such that :

$$L([1,0,0]) = [0,-2,1], \quad L([0,1,0]) = [-1,1,2], \\ L([0,0,1]) = [2,3,-3].$$

Find $L([2, -3, 1])$. Give a formula for $L([x, y, z])$ for any $[x, y, z] \in \mathbb{R}^3$.

- (c) Let $L: M_{3 \times 3} \rightarrow \mathbb{R}$ be a linear transformation given by $L(A) = \text{trace}(A)$. Find a basis for $\ker(L)$ and $\text{range}(L)$. Also, verify that $\dim(\ker(L)) + \dim(\text{range}(L)) = \dim(M_{3 \times 3})$.

6. (a) Let $L: \mathbb{R}^3 \rightarrow \mathbb{R}^2$ be a linear transformation given by :

- (b) Express the vector $x \in \mathbb{R}^3$ as a linear combination of the other vectors (if possible)

$$x = [5, 9, 5], \quad a_1 = [2, 1, 4], \quad a_2 = [1, -1, 3], \quad a_3 = [3, 2, 5].$$

- (c) Show that the vector $x = [5, 17, 20]$ is in the row space of the given matrix

$$A = \begin{bmatrix} 3 & 1 & -2 \\ 4 & 0 & 1 \\ -2 & 4 & -3 \end{bmatrix}.$$

3. (a) Find the characteristic polynomial and eigen values of the matrix :

$$A = \begin{bmatrix} 4 & 0 & -2 \\ 6 & 2 & -6 \\ -2 & 0 & 1 \end{bmatrix}.$$

Find the eigen spaces corresponding to the eigen values of A .

6081

4

- (b) Let $V = \mathbb{R}^2$ be a vector space with respect to the operations of addition and scalar multiplication, defined as, for $[x, y], [z, w] \in V$ and $a \in \mathbb{R}$:

$$[x, y] \oplus [z, w] = [x + w - 2, y + z + 3] \text{ and } a \odot [x, y] = [ax - 2a + 2, ay - 3a - 3].$$

Prove the closure and associative properties with respect to the addition operation in V . Also, find the zero vector and the additive inverse of each vector in V .

- (c) Consider the subset $S = \{[3, 1, -1], [5, 2, -2], [2, 2, -1]\} \subset \mathbb{R}^3$. Use the simplified Span method to find the $\text{span}(S)$. Also, find the dimension of $\text{span}(S)$.

4. (a) Use the Independence test method to determine whether the given set

$$S = \{[2, 5, -1, 6], [4, 3, 1, 4], [1, -1, 1, -1]\} \subset \mathbb{R}^4$$

is linearly independent or linearly dependent in $\mathbb{R}^4$.

6081

5

- (b) (i) Check whether the set

$$S = \left\{ a \begin{bmatrix} a & -a \\ b & 0 \end{bmatrix} \in M_{2 \times 2} \mid a, b \in \mathbb{R} \right\}$$

is a subspace of $M_{2 \times 2}$ or not, under usual matrix operations.

- (ii) Check whether the set $S = \{[1, a] \in \mathbb{R}^2 \mid a \in \mathbb{R}\}$ is a subspace of $\mathbb{R}^2$ or not, under usual operations.

- (c) Prove that the set

$$S = \left\{ \begin{bmatrix} 2 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & -1 \\ 2 & -2 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix}, \begin{bmatrix} 3 & 1 \\ 0 & -2 \end{bmatrix} \right\}$$

is a basis for $M_{2 \times 2}$.

5. (a) Determine whether the following functions are linear transformations or not:

$$(i) L_1: \mathbb{R}^3 \rightarrow \mathbb{R}^3 \text{ given by } L_1([x_1, x_2, x_3]) = [5x_1 - x_3, x_2, 3x_3 - x_2].$$