(c) Let $L : P_2 \rightarrow P_3$ be a linear transformation such that

$$L(1) = 1,\ L(x) = x^2,\ L(x^2) = x^3 + x.$$

Find $L(2x^2 - 5x + 3)$. Give a formula for $L(ax^2 + bx + c)$.

[This question paper contains 8 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : 7714 **H**

Unique Paper Code : 2354001202

Name of the Paper : Introduction to Linear Algebra

Name of the Course : **GE II**

Student : SOL

Semester : II

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. Attempt **all** questions by selecting **two** parts from each question.

3. **All** questions carry equal marks.

1. (a) Find a point that is two-thirds of the distance from the initial point P = (2, –1, 0, –7) to the terminal point Q = (–11, –1, –9, 2). Also, find the norm of the vector $\overrightarrow{PQ}$.

(b) Solve the following system of linear equations by using Gaussian elimination method :

$$3x - 2y + 4z = -54$$

$$-x + y - 2z = 20$$

$$5x - 4y + 8z = -83.$$

Also, indicate whether the system is consistent or inconsistent.

(c) Without using the Cauchy-Schwarz inequality, show that if a and b are unit vectors in $\mathbb{R}^n$, then $|a . b| \leq 1$.

6. (a) Let L : $\mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation given by :

$$L\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} 1 & 2 & 7 \\ 2 & 0 & -6 \\ -1 & 5 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}.$$

(i) Is [–3, 2, 1] in ker(L)? Justify the answer.

(ii) Is [6, –4, –5] in range(L)? Justify the answer.

(b) Let L: $M_{2\times2} \to \mathbb{R}^2$ be a linear transformation given by :

$$L\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = [6a - b + 3c - 2d,\ -2a + 3b - c + 4d].$$

Find the matrix representation for L with respect to the standard basis for $M_{2\times2}$ and $\mathbb{R}^2$ respectively. Use this matrix to calculate $L\left(\begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}\right)$ by matrix multiplication.

(i) $L_1 : \mathbb{R}^3 \to \mathbb{R}^3$ given by $L_1([x_1, x_2, x_3]) = [3x_1, -x_2, x_3 + 2x_2]$.

(ii) $L_2 : P_2 \to \mathbb{R}$ given by $L_2(ax^2 + bx + c) = abc$.

(b) Let $L : M_{2\times3} \to M_{2\times2}$ be a linear transformation given by :

$$L\left(\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}\right) = \begin{bmatrix} a & -c \\ 2e & d+f \end{bmatrix}.$$

Determine if L is one-one or not, and onto or not.

(c) Let $L : P_2 \to P_3$ be a linear transformation given by :

$$L(ax^2 + bx + c) = cx^3 + bx^2 + ax.$$

Find a basis for ker(L) and range(L). Also, verify that $\dim(\ker(L)) + \dim(\text{range}(L)) = \dim(P_2)$.

2. (a) Determine whether the vector $x = [7,1,8]$ is in the row space of the given matrix

$$A = \begin{bmatrix} 3 & 6 & 2 \\ 2 & 10 & -4 \\ 2 & -1 & 4 \end{bmatrix}.$$

(b) Express the vector $x \in \mathbb{R}^3$ as a linear combination of the other vectors (if possible)

$x = [2, 2, 3]$, $a_1 = [6, -2, 3]$, $a_2 = [0, -5, -1]$, $a_3 = [-2, 1, 2]$.

(c) Solve the following homogeneous system of linear equations by using the Gauss-Jordan row reduction method and express the complete solution set as a linear combination of particular solutions

$$-2x + y + 8z = 0$$

$$7x - 2y - 22z = 0$$

$$3x - y - 10z = 0.$$

3. (a) Check whether the following matrix is diagonalizable or not :

$$\begin{bmatrix} 1 & 3 & -3 \\ -3 & -5 & -3 \\ 3 & 3 & 1 \end{bmatrix}.$$

(b) Let V be a vector space. Prove the following:

(i) If u, v and w are vectors in V for which u + w = v + w, then u = v.

(ii) $0.v = 0$ and $(-1).v = -v, \ \forall \ v \in V$.

(c) Consider the subset $S = \{[1, 4, 1, -2], [-1, 1, 1, -1], [3, 2, -1, 0], [2, 3, 0, -1]\} \subset \mathbb{R}^4$. Use the simplified Span method to find the span(S). Does span(S) = $\mathbb{R}^4$? Justify.

4. (a) Use the Independence test method to determine whether the set

$$S = \left\{ \begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix}, \begin{bmatrix} -1 & 0 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 6 & -1 \\ 3 & 2 \end{bmatrix}, \begin{bmatrix} -11 & 3 \\ -2 & 2 \end{bmatrix} \right\}$$

is linearly independent or linearly dependent.

(b) (i) Check whether the set of all singular matrices in $M_{2\times2}$ is a subspace of $M_{2\times2}$ or not, under the usual matrix operations.

(ii) Show that the set $S = \{[a, b, a - 2b] \in \mathbb{R}^3 \mid a, b \in \mathbb{R}\}$ is a subspace of $\mathbb{R}^3$, under the usual operations.

(c) Find the basis and dimension for the subspace W of $\mathbb{R}^3$ spanned by the set

$$S = \{[3, 2, -1], [1, 2, 0], [1, 2, -1]\}.$$

5. (a) Determine whether the following functions are linear transformations or not :