(c) Let L: $\mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation such that :

L([1,0,0])= [1,1,2], L([0,1,0]) = [0,1,1], L([0,0,1]) = [1,2,3],

Find L([5,1,–3]). Give the formula for L([x,y,z]) for any [x,y,z] $\in \mathbb{R}^3$.

[This question paper contains 8 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 5662 **H**

Unique Paper Code : 2354001202

Name of the Paper : Introduction to Linear Algebra

Name of the Course : **GE II**

Student : SOL

Semester : II

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. Attempt **all** questions by selecting **two** parts from each question.

3. **All** questions carry equal marks.

1. (a) State the Cauchy- Schwarz inequality in $\mathbb{R}^n$, $n \geq 2$ and verify the same for the vectors

$$x = [-1, 4, 2, 0, -3], \quad y = [2, 1, -4, -1, 0].$$

(b) Find a unit vector in the same direction as the given vector $\vec{a} = \left[\frac{5}{6}, \frac{-2}{3}, \frac{3}{4}, 0, \frac{-5}{2}\right]$. Is the resulting vector longer or shorter than the original? Justify your answer. Also, find a vector of length 5 in the negative direction of the given vector $\vec{a}$.

(c) Solve the following system of linear equations by using Gaussian elimination method :

$$-5x - 2y + 2z = 16$$

$$3x + y - z = -9$$

$$2x + 2y - z = -4.$$

$$L(ax^2 + bx + c) = \begin{bmatrix} a+c & 0 \\ b-c & -3a \end{bmatrix}.$$

Determine whether L is one-one or not, and onto or not.

6. (a) Let $L: \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation given by :

$$L\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} 2 & 4 & 6 \\ -3 & 6 & 1 \\ 8 & 16 & 24 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}.$$

(i) Is [−8, −5, 6] in ker(L)? Justify the answer.

(ii) Is [0, 2, 0] in range(L)? Justify the answer.

(b) Let $L : P_2 \to \mathbb{R}$ be a linear transformation given by $L(p) = p(1)$. Find a basis for ker(L) and range(L). Also, verify that dim(ker(L)) + dim(range(L)) = dim(P_2).

5. (a) Determine whether the following functions are linear transformations or not :

(i) $L_1 : \mathbb{R}^3 \to \mathbb{R}^3$ given by $L_1([x_1, x_2, x_3]) = [x_1 + 1,\ 2 - x_2, x_3]$.

(ii) $L_2 : M_{2\times 2} \to \mathbb{R}$ given by $L_2\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = ad - bc$.

(b) Let $L : P_3 \to P_2$ be a linear transformation given by $L(p) = p'$, for $p \in P_3$. Find the matrix representation for L with respect to the standard bases for P_3 and P_2 respectively. Use this matrix to calculate $L(8x^3 + 7x^2 - 3x + 4)$ by matrix multiplication.

(c) Let $L : P_2 \to M_{2\times 2}$ be a linear transformation given by :

2. (a) Solve the system of linear equations, whose augumented matrix is given below, by using Gauss-Jordan row reduction method :

$$\left[\begin{array}{cccc|c} 2 & 1 & 3 & 0 & 16 \\ 3 & 2 & 0 & 1 & 16 \\ 2 & 0 & 12 & -5 & 5 \end{array}\right].$$

(b) Find the rank of the following matrix :

$$\begin{bmatrix} -1 & -1 & 0 & 0 \\ 0 & 0 & 2 & 3 \\ 4 & 0 & -2 & 1 \end{bmatrix}.$$

(c) Express the vector $x \in \mathbb{R}^3$ as a linear combination of the other vectors (if possible)

$x = [7,2,3]$, $a_1 = [1,-2,3]$, $a_2 = [5,-2,6]$, $a_3 = [4,0,3]$.

3. (a) Find the characteristic polynomial of the matrix

$$A=\begin{bmatrix}12 & -51\\ 2 & -11\end{bmatrix}.$$

Also, find all the eigen values and eigen spaces for A.

(b) Use the Independence test method to determine whether the given set

$S = \{3x^2 + x - 1, -5x^2 - 2x + 2, 2x^2 + 2x - 1\}$

is linearly independent or linearly dependent in P_2.

(c) Show that the set $S = \{[1,-2,-2], [3,-5,1], [-1,1,-5]\}$ does not span $\mathbb{R}^3$.

4. (a) Let $V = \mathbb{R}^2$ be a vector space with respect to the operations of addition and scalar multiplication, defined as, for $[x,y], [z,w] \in V$ and $a \in \mathbb{R}$:

$[x, y] \oplus [w, z] = [x + w - 2, y + z + 3]$ and

$a \odot [x, y] = [ax - 2a + 2, ay - 3a - 3]$.

Prove the closure properties with respect to the above operations in V and also prove the associative property with respect to the addition operation in V. Find the zero vector and the additive inverse of each vector in V.

(b) (i) Check whether the set $S = \{[3, a] \in \mathbb{R}^2 \mid a \in \mathbb{R}\}$ is a subspace of $\mathbb{R}^2$ or not, under the usual operations.

(ii) Show that the set $S = \{[4a + 2b, a, b] \in \mathbb{R}^3 \mid a, b \in \mathbb{R}\}$ forms a subspace of $\mathbb{R}^3$, under the usual operations.

(c) Determine whether the set

$B = \{[1,4,2,0], [0,2,1,0], [-3,1,-1,0],[5,-2,0,-3]\}$

forms a basis for $\mathbb{R}^4$ or not.