(c) (i) Determine whether the geometric series

$$\sum_{n=1}^{\infty} \frac{3^n}{4^{n+2}}$$

converges or diverges. Find the sum of the series if it converges.

(ii) Show that 0.002002002 ..... converges and find it's sum.

6. (a) State Integral Test. Using this test or otherwise, test the convergence of the following series :

(i) $\sum_{n=1}^{\infty} \frac{n}{n^2+10}$

(ii) $\sum_{n=1}^{\infty} \frac{1}{n^3}$

(b) Test the convergence of following series :

(i) $\sum_{n=2}^{\infty} \frac{1}{n!}$

(ii) $\sum_{n=1}^{\infty} \left(\frac{1}{3}+\frac{1}{n}\right)^n$

(c) (i) Show that the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2n-1}$ converges using Alternating Series Test.

(ii) Test the convergence and absolute convergence of the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^p}$; for $p>0$.

[This question paper contains 4 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 5873 **H**

Unique Paper Code : 2354002003

Name of the Paper : Elements of Real Analysis

Name of the Course : **COMMON PROG GROUP**

Semester : IV

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **all** question by selecting **two** parts from each question.
3. Part of the questions to be attempted together.
4. **All** questions carry equal marks.
5. Use of Simple Calculator allowed.

1. (a) Let $a<b$ in an ordered field F. Then prove that $b = \sup(a, b)$.

(b) Prove that any complete ordered field is Archimedean.

(c) Prove that for all x, y $\in$ ordered field F, $\max\{x, y\}$

$$= \frac{x+y+|x-y|}{2} \text{ and } \min\{x,y\} = \frac{x+y-|x-y|}{2}.$$

2. (a) Prove that in an ordered field F, u is a lower bound of a set A if and only if $-u$ is an upper bound of the set $-A = \{-a : a \in A\}$.

(b) Verily whether all the field axioms for the set $\mathbb{R}^2$ with $(x, y) + (u, v) = (x + u, y + v)$ and $(x, y).(u, v) = (xu, yv)$ are satisfied or not.

(c) Define a convergent sequence. Using the definition of convergence, show that

$$\lim_{n\to\infty} \frac{3n^2+n-5}{n^2+6n} = 3.$$

3. (a) Prove that every monotonically increasing bounded above sequence is convergent.

(b) Let $X = \langle x_n \rangle$ and $Y = \langle y_n \rangle$ be sequences of real numbers that converges to x and y, respectively, then prove that the sequence $X - Y$ converges to $x - y$.

(c) Prove that the sequence $\langle x_n \rangle = \langle (-1)^n \rangle$ is not a cauchy sequence.

4. (a) Let $\langle x_n \rangle$ be a sequence of real numbers defined inductively by $x_1 = 1$ and $x_{n+1} = \sqrt{4x_n+5}$, for all $n \in N$. Prove that $\langle x_n \rangle$ converges and find its limit.

(b) State second squeeze principle for convergence of sequences and use it to prove that $\lim\limits_{n\to\infty} \frac{2n+3}{3n-7} = \frac{2}{3}$.

(c) Suppose that $\langle x_n \rangle$ is a sequence of real numbers for which there is a constant $c > 0$ such that for all $n \in N$, $|x_{n+1} - x_n| < \frac{C}{2^n}$. Then prove that $\langle x_n \rangle$ is a cauchy sequence.

5. (a) Show that the nth partial sum of series

$\sum_{n=0}^{\infty} ar^n = a + ar + ar^2 + \cdots (a \neq 0)$ is $\frac{a}{1-r}(1-r^n)$, if $r \neq +1$. Hence, show that the geometric series converges to $\frac{a}{(1-r)}$ if $|r| < 1$.

(b) State and prove Cauchy Criterion for convergence of series.