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SET - 2

Serial Number of Question Paper	:	**5894**	Your Roll No............
Unique Paper Code	:	2224002006	
Name of the Paper	:	Quantum Mechanics	
Name of the Course	:	B.Sc. Hons._NEP: UGCF-2022_GE	
Semester	:	IV	
Duration: 3Hours		Maximum Marks: 90	

(Write your Roll No. on the top immediately on receipt of this question paper)

Attempt only five (5) questions.

Question No. 1 is compulsory.

All questions carry equal marks.

Use of non-programmable scientific calculator is allowed.

1. Attempt all the questions. $6 \times 3 = 18$

   (i) Which of the following wave functions are physically acceptable? Justify your answer (a) $1/x^4$ (b) $\exp(-2x)$.

   (ii) How quantum description of Harmonic oscillator is different from the classical description?

   (iii) Consider an electron confined to a one-dimensional infinite potential well of width L=1.0 nm. Calculate the energies of first two excited states.

   (iv) Explain the concept of zero-point energy.

   (v) State Ehrenfest's theorem.

   (vi) Verify the operator equation
   $$x^2p - px^2 = 2i\hbar x$$

2.

   (i) An electron is moving freely inside a one-dimensional infinite potential box with walls at x = 0 and x = a. If the electron is initially in the ground state (n = 1) of the box and if the size of the box quadruples suddenly (the right-hand wall is moved instantaneously from x = a to x = 4a), calculate the probability of finding the electron in the ground state and first excited state of the new box.

   (ii) Normalize the wave function:
   $$\Psi(x) = e^{-|x|} \sin 2x$$
   (10, 8)

3.

(i) Explain the spreading of a Gaussian wave-packet with time.

(ii) Find $\psi(x)$ for a Gaussian wave packet $\phi(k) = Aexp[-a^2(k-k_0)^2/4]$, where A is normalization constant. Calculate the probability to finding the particle in the region $-a/4 \leq x \leq a/4$. (10, 8)

4.

(i) Defining the raising and lowering operators as $\hat{L}_+ = \hat{L}_x + i\hat{L}_y$ and $\hat{L}_- = \hat{L}_x - i\hat{L}_y$ respectively, show that $[\widehat{L_z}, \hat{L}_+] = i\hbar\hat{L}_+$ and $[\hat{L}_+, \hat{L}_-] = 2\hbar\widehat{L_z}$.

(ii) If the incident beam has an energy of 15 eV, V = 10 eV and a = 1Å, calculate the order of magnitude values of the transmission and reflection coefficients. (10, 8)

5.

(i) Solve the time independent Schrodinger equation for the energy levels of a 1-D harmonic oscillator.

(ii) A harmonic oscillator has a wavefunction which is a superposition of its ground state and first excited state eigenfunctions, i.e.,

$$\psi(x) = \frac{1}{\sqrt{2}}[\psi_0(x) + \psi_1(x)]$$

Find the expectation value of the energy.

(13, 5)

6. (i) Starting from the Schrodinger equation for hydrogen atom in spherical polar coordinates, split the equation into three parts.

(ii) The wave function of the Hydrogen atom in 1s state is given by $\psi(1s) = \frac{1}{\sqrt{\pi}(a_0)^{3/2}}\exp(-\frac{r}{a_0})$. Calculate the expectation value of the potential energy of the electron in this state. (10, 8)