(c) Show that the mapping f: $M_{nm} \rightarrow M_{mn}$ defined as $f(A) = A^T$ is an isomorphism. (7.5)

[This question paper contains 8 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 4020 **H**

Unique Paper Code : 2352571201

Name of the Paper : Elementary Linear Algebra

Name of the Course : **B.A. / B.Sc. (Prog.) with Mathematics as Non-Major/ Minor – DSC**

Semester : II

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **all** questions by selecting any **two** parts from each question.
3. **All** questions carry equal marks.
4. Use of simple calculator is allowed.

1 (a) If x and y are vectors in $\mathbb{R}^n$, then prove that $||x + y|| \geq |\,||x|| - ||y||\,|$. Also verify it for the vectors $x = [2, -1, 3, 2]$ and $y = [4, 3, 2\ 1]$ in $\mathbb{R}^4$. (5.5+2)

(b) Solve the following system of linear equations using Gaussian Elimination method. Also indicate whether the system is consistent or inconsistent.

$$3x - 2y + 4z = -54$$

$$-x + y - 2z = 20$$

$$5x - 4y + 8z = -83$$

(6.5+1)

(c) Find the quadratic equation $y = ax^2 + bx + c$ that goes through the points (3,18), (2,9) and (–2,13). (7.5)

(b) Find the matrix T_{AB} of linear transformation T: $R^2 \rightarrow R^3$ with respect to basis B = {(1,0), (0,1)} and C = {(1,1,0), (0,1,1), (1,0,1)}, where

$$T([a,b]) = [a - b, a, 2a + b]$$

(7.5)

(c) Define kernel of a linear transformation T. Show that Ker(T) = {0} if and only if T is one-one. (2+5.5)

6. (a) Consider the linear operator $L : M_{33} \rightarrow M_{33}$, defined as $L(A) = A - A^T$, where M_{33} is vector space of 3×3 matrices and A^T denotes the transpose of the matrix A.

Find dim(Ker(L)) + dim(Range(L)). (7.5)

(b) Define an onto linear transformation. If T be a linear transformation from a finite dimensional vector space V to a finite dimensional vector space W, show that T is onto if and only if dim (Range(L)) = dim (W). (2+5.5) where $A=\begin{pmatrix} 1 & -2 & 1 \\ 0 & 1 & 1 \end{pmatrix}$. Show the following :

(i) W is a subspace of R^3.

(ii) Find a basis for W. (1.5+3+3)

5. (a) Check if the following mappings are linear transformation or not. Prove it or give a counter example to disprove.

(i) f: $R^3 \rightarrow R^3$ defined as $f([a, b, c]) = [-b, c, 0]$

(ii) g: $M_{22} \rightarrow R$ defined as $g\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = ad - bc$,

where M_{22} is the vector space of 2×2 real matrices. (4+3.5)

2. (a) Solve the following system of equations using the Gauss-Jordan method :

$$5x + 20y - 18z = -11$$

$$3x + 12y - 14z = 3$$

$$-4x - 16y + 13z = 13$$ (7.5)

(b) Find the rank of the following matrix by converting to row echelon form :

$$\begin{bmatrix} 8 & 0 & 0 & 1 \\ 1 & 0 & 3 & 1 \\ 0 & 0 & 1 & 3 \\ 0 & 8 & 1 & 8 \end{bmatrix}$$ (7.5)

(c) Find the characteristic polynomial, eigenvalues and corresponding eigenvectors for the given matrix :

$$\begin{bmatrix} -1 & 2 \\ 4 & -3 \end{bmatrix}$$ (2+2+3.5)

3. (a) Use the Diagonalization Method to determine whether the following matrix is diagonalizable.

$$\begin{bmatrix} 3 & 1 & -1 \\ 2 & 2 & -1 \\ 2 & 2 & 0 \end{bmatrix}$$ (7.5)

(b) Show that the set M_{22} of all 2×2 matrices is a vector space under the usual operations of addition and scalar multiplication. (7.5)

(c) Show that span (S) is a subspace of V, where S a nonempty subset of a vector space V. Let P_3 be the vector space of all real polynomials of degree ≤ 3 and $S = \{x, x^2 + 1, x^3 - 1\}$. Find span (S). Does $(x^2 + x^3) \in$ span (S)? (3+3+1.5)

4. (a) Check whether the following subset of R is linearly independent or not.

$$S = \{(0,1,-1), (1,1,0), (1,0,2)\}.$$

Express (1,1,1) as linear combination of vectors in S. (4+3.5)

(b) Define a basis of a vector space. Show that the following set S is a basis for the vector space M_{22} of all 2×2 matrices :

$$S = \left\{ \begin{pmatrix} 1 & 4 \\ 2 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 2 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} -3 & 1 \\ -1 & 0 \end{pmatrix}, \begin{pmatrix} 5 & -2 \\ 0 & -3 \end{pmatrix} \right\}$$ (2+5.5)

(c) Define a finite dimensional vector space. Let W be the solution set to the matrix equation $AX = 0$,