Unit III

5. (a) Find all the asymptotes of the curve

$2x^3 + 3x^2y - 3xy^2 - 2y^3 + 3x^2 - 3y^2 + y = 3$. (7.5)

(b) (i) Find the points of inflexion of the curve

$y = 3x^4 - 4x^3 + 1$. (3)

(ii) Find the range of values of x in which the curve $y = 3x^5 - 40x^3 + 3x - 20$ is concave upwards and concave downwards. (4.5)

(c) Trace the curve $x^3 + y^3 = a^2x$. (7.5)

6. (a) Prove that at $x = a$ the curve $ay^2 = (x-a)^2(x-b)$ has a conjugate point if $a < b$, a node if $a > b$, a cusp if $a = b$. (7.5)

(b) Using the result

$$\int \sin^n x dx = -\frac{\cos x \sin^{n-1} x}{x} + \frac{n-1}{n}\int \sin^{n-2} x dx,$$

evaluate the definite integral $\int_0^{\pi/2} \sin^n x dx$, where n is a positive integer and $n \geq 2$. (7.5)

(c) Trace the curve $r = a \cos 3\theta$. (7.5)

[This question paper contains 4 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 1008 **D**

Unique Paper Code : 2352571101

Name of the Paper : Topics in Calculus

Name of the Course : **BA / B.Sc. (Prog.) with Mathematics as Non-Major/ Minor**

Semester : I

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. This question paper has **six** questions in all.
3. Attempt any **two** parts from each question.
4. **All** questions are compulsory.

Unit I

1. (a) Use (ε, δ) definition of a limit to prove that $\lim_{x\to3}(3x - 7) = 2$. (7.5)

P.T.O.

(b) Show that the function f(x) defined as $f(x) = |x-1| + |x+1|$, $x \in R$ is not derivable at the point $x = -1$ and $x = 1$, is derivable at every other point. (7.5)

(c) Discuss the continuity of $f(x) = [x]$ at $x = 1$.

Also mention the type of discontinuity. (7.5)

2. (a) If $y = \frac{\sin^{-1} x}{\sqrt{1-x^2}}$, prove that

$$(1-x^2)y_{n+1} - (2n+1)xy_n - n^2y_{n-1} = 0.$$ (7.5)

(b) (i) Find $\frac{d^2y}{dx^2}$, if $x = a(\cos\theta + \theta\sin\theta)$,

$y = a(\sin\theta - \theta\cos\theta)$.

(ii) If $z = x^y$, find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$. (7.5)

(c) If $u = \log\left(\frac{x^4+y^4}{x+y}\right)$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 3$.

(7.5)

Unit II

3. (a) State and prove Rolle's theorem. (7.5)

(b) What is the geometrical interpretation of Lagrange's mean value theorem. Find 'c' of Lagrange's mean value theorem for the following function:

$f(x) = x(x-1)(x-2)$; $a = 0$, $b = \frac{1}{2}$. (7.5)

(c) State the Cauchy mean value theorem. Find the value of 'c' of Cauchy's mean value theorem for the following functions

$f(x) = \sin x$ and $g(x) = \cos x$ in $[-\pi/2, 0]$ (7.5)

4. (a) State Taylor's theorem. Hence expand cosx in terms of $(x - \pi/4)$. (7.5)

(b) Assuming $R_n(x) \to 0$ as $n \to \infty$, find the Maclaurin's series expansion of $\sin(x/2)$. (7.5)

(c) (i) Evaluate $\lim_{x\to 0}\left(\frac{1}{x} - \frac{1}{\sin x}\right)$. (3)

(ii) For what value of a does $\frac{\sin 2x - a\sin x}{x^3}$ tend to a finite limit as $x \to 0$? (4.5)