(b) Find the m.g.f of the normal distribution $N(\mu, \sigma^2)$ and deduce that

$\mu_{2n+1} = 0$ for every $n = 0, 1, 2, \ldots$ and

$\mu_{2n} = 1.3.5 \ldots (2n - 1)\sigma^{2n}$, for every $n = 1, 2, \ldots$

where, μ_n is the n^{th} central moment. (7,8)

7\. (a) Define exponential distribution and show that it 'Lacks Memory'.

(b) Obtain the m.g.f. of gamma distribution with parameter λ. Also, show that the sum of two independent gamma variates is also a gamma variate. (7,8)

8\. (a) In a binomial distribution consisting of 5 independent trials, probabilities of 1 and 2 successes are 0.40 and 0.20 respectively. Find the parameter p of the distribution.

(b) Find the expectation of the number on a die when thrown.

(c) If X and Y are independent Poisson variates such that

$P(X = 1) = P(X = 2)$ and
$P(X = 2) = P(X = 3)$.

Find the variance of $(X - 2Y)$. (5,5,5)

[This question paper contains 4 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 4921 **H**

Unique Paper Code : 237457201

Name of the Paper : Statistical Methods

Name of the Course : **B.A. / B.Sc. (Prog.) – NEP Minor**

Semester : II (NEP)

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **six** questions in all.

1\. (a) A random variable X has the probability function:

x	0	1	2	3	4	5	6	7
p(x)	1	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$k^2 + k$

Find (i) Value of k, (ii) $P(X \geq 6)$ and (iii) The distribution function of X.

(b) The p.d.f. of a continuous random variable X is given by :

$$f(x) = \begin{cases} \frac{1}{3}, & -1 \leq x \leq 2 \\ 0, & \text{elsewhere.} \end{cases}$$

Obtain m.g.f. of X. Hence, find mean and variance of X. (7,8)

2. (a) The joint p.d.f of a two dimensional continuous random variable (X, Y) is given by

$$f(x,y) = \begin{cases} \frac{x^3y^3}{16}, & 0 \leq x \leq 2, 0 \leq y \leq 2 \\ 0, & \text{elsewhere.} \end{cases}$$

(i) Find the marginal p.d.f. of X

(ii) Conditional p.d.f. of Y given X.

(b) State and prove multiplication theorem of expectation for two independent random variables. If X and Y are independent random variables with means 10 and 20 and variances 2 and 3 respectivly. Find the expectation and variance of the variable 3X + AY. (7,8)

3. (a) If X_1 and X_2 are random variables, then show that

$V(aX_1 + bX_2) = a^2Var(X_1) + b^2Var(X_2) + 2ab\,Cov(X_1,X_2)$.

Also, discuss the case when X_1 and X_2 are independent.

(b) Define characteristic function of a random variable X. If X_1 and X_2 are independent random variables, then show that :

$$\phi_{X_2+X_2}(t) = \phi_{X_1}(t).\phi_{X_2}(t).$$ (7,8)

4. (a) Define binomial distribution. Obtain the moment generating function of a binomial distribution. Hence, find its mean, variance and third order moment.

(b) If $X \sim B(n, p)$, show that :

$$E\left(\frac{X}{n}-p\right)^2 = \frac{pq}{n};\ \text{cov}\left(\frac{X}{n}, \frac{n-X}{n}\right) = -\frac{pq}{n}.$$ (7,8)

5. (a) Obtain the mode of Poisson distribution with mean λ. Hence find the mode in case when (i) $\lambda = 3.5$, and (ii) $\lambda = 5$.

(b) State and prove addition property of Poisson distribution. (7,8)

6. (a) If $X \sim N(\mu, \sigma^2)$, find its median and mode. If $X \sim N(5, 9)$, what will be its mean, mode and variance?