$$\mu_{r+1} = r\lambda\mu_{r-1} + \lambda\frac{d\mu_r}{d\lambda}$$

where, μ_r is the r^{th} moment about the mean. Hence obtain the skewness and kurtosis of Poisson distribution. (8,7)

6. (a) If X is a gamma variate with parameter λ, obtain its cumulant generating function.

   (b) If $X \sim Expo(\lambda)$ with $P(X \leq 1) = P(X > 1)$, then find variance of X. (8,7)

7. (a) Show that a linear combination of independent normal variates is also a normal variate.

   (b) Find the moment generating function of the normal distribution with parameters μ and σ^2. (8,7)

8. (a) Show that mode for normal distribution coincide with mean.

   (b) A continuous random variable has the following probability density function:

   $$f(x) = A x^2, \ 0 \leq x \leq 1.$$

   Determine the value of (i) A, (ii) $P(0.2 \leq X \leq 0.5)$, and (iii) $P\left(X > \frac{3}{4} \middle| X > \frac{1}{2}\right)$. (8,7)

[This question paper contains 4 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 4015 **H**

Unique Paper Code : 2374571201

Name of the Paper : Statistical Methods

Name of the Course : **B.A. (Prog.) / B.Sc. (Prog.) with Statistics (NEP-UGCF)**

Semester : II

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **Six** questions in all.
3. Each question carry equal marks.
4. Simple calculator is allowed.

1. (a) A random variable X has the following probability mass function :

x	0	1	2	3	4	5	6	7
P(x)	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2+k$

   Find k, $P(0 < X < 4)$ and E(X).

(b) The two-dimensional continuous random variable (X, Y) has joint pdf.

$$f(x,y) = \begin{cases} e^{-(x+y)}; & x \geq 0, y \geq 0 \\ 0 & ; \text{elsewhere} \end{cases}$$

Find

(i) Marginal pdf of X

(ii) $P(X > 1)$

(ii) $P(X > 2, Y > 4)$. (8,7)

2. (a) The joint probability distribution of X and Y is given as follows.

x \ y	1	3	9
1	k	k	k
2	k	$2k$	k
3	k	k	k

Find k, marginal distributions of X and Y. Also find E(X).

(b) State and prove additive property of expectation for two random variables. Hence, find mean and variance of the random variable $Y = 2X_1 - 3X_2$, where X_1 and X_2 are two independent random variables with mean 3 and 5, and variance 5 and 6 respectively. (8,7)

3. (a) A random variable X has the probability function :

$$p(x) = \frac{1}{2^x}; \; x = 1, 2, 3, \cdots$$

Obtain the m.g.f. of X and hence find the mean and variance of X.

(b) Define the characteristic function $\varphi_X(t)$ of a random variable X. If X and Y are independent random variables then prove that

$\phi_{X+Y}(t) = \phi_X(t) . \phi_Y(t)$. (8,7)

4. (a) If two independent variables X_1 and X_2 have Poisson distribution with means λ_1 and λ_2 respectively, then show that $X_1 + X_2$ is also a Poisson variate with mean $\lambda_1 + \lambda_2$.

(b) If X follows Poisson distribution with parameter λ, then find its β_1 and β_2 coefficients. (8,7)

5. (a) If X follows binomial distribution with parameters n and p, find its mode. If $X_1 \sim B\left(8, \frac{1}{3}\right)$ and $X_2 \sim B\left(14, \frac{1}{3}\right)$, find the mode of X_1 and X_2.

(b) Show that the recurrence relation between moments of Poisson distribution is given by