(b) Find the matrix for linear operator L: $\mathbb{R}^3 \to \mathbb{R}^3$ defined as

$$L([x_1, x_2, x_3]) = [3x_1 + x_2, x_1 + x_2, x_1 - x_2]$$

with respect to the standard basis $B = (e_1, e_2, e_3)$ for $\mathbb{R}^3$.

(c) Let L: $P_3(x) \to P_2(x)$ be a linear transformation defined as $L(p(x)) = p'(x)$ where $p'(x)$ is the derivative of p(x) and $P_n(x)$ is the vector space of all polynomials of degree atmost n. Find the kernel of L.

6. (a) Let L: $\mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation defined as

$$L\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} 1 & -1 & 5 \\ -2 & 3 & -13 \\ 3 & -3 & 15 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}.$$

Find the basis for kernel of L.

(b) Show that the linear operator L: $\mathbb{R}^2 \to \mathbb{R}^3$ defined as

$$L([x,y]) = [x, y, x + y]$$

is one-to-one but not onto.

(c) Prove that the linear transformation L: $P_n(x) \to P_n(x)$ defined as $L(p(x)) = p(x) + p'(x)$ where $p'(x)$ is the derivative of p(x) and $P_n(x)$ is the vector space of all polynomials of degree atmost n is an isomorphism.

[This question paper contains 4 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 5532 **H**

Unique Paper Code : 2352571201

Name of the Paper : Elementary Linear Algebra

Name of the Course : **B.A. / B.Sc. (Prog.) with Mathematics as Non-Major/ Minor – DSC**

Semester : II

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** questions are compulsory.
3. Questions **1** to **6** have three parts each. Attempt any **TWO** parts from each Question. Each part carries **7.5** marks.
4. Use of Calculator not allowed.

1. (a) State and prove Triangle Inequality.

(b) Define rank of a matrix. Find the rank of the matrix

$$\begin{bmatrix} 2 & 1 & 4 \\ 3 & 2 & 5 \\ 0 & -1 & 1 \end{bmatrix}.$$

(c) Using Gaussian Elimination method, find the quadratic equation $y = ax^2 + bx + c$ that goes through the points (2,9), (3,18) and (–2,13).

2. (a) Verify Cauchy-Schwarz Inequality for the vectors $[-1,-1,0]$ and $\left[\sqrt{2},\sqrt{2},\sqrt{2}\right]$ in $\mathbb{R}^3$.

(b) Let $x = [7,2,3] \in \mathbb{R}^3$ and $S = \{[1, -2, 3], [5, -2, 6], [4, 0, 3]\}$ be a subset of $\mathbb{R}^3$. Is it possible to express x as a linear combination of elements of set S? If yes, then find the expression.

(c) Using rank, determine whether the following homogeneous system of linear equations has a non-trivial solution or not

$$\begin{aligned} 2x_1 + x_2 + 4x_3 &= 0, \\ 3x_1 + 2x_2 + 5x_3 &= 0, \\ -3x_2 + x_3 &= 0. \end{aligned}$$

3. (a) Determine whether the following matrix is diagonalizable. If yes, specify the matrices D and P such that $D = P^{-1}AP$

$$A = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}.$$

(b) Show that set of all polynomials P(x) forms a vector space under usual polynomial addition and scalar multiplication.

(c) Define subspace of a vector space V. Let A be a $n \times n$ matrix and let λ be the eigen value of A. Prove that the eigen space of λ, E_λ is a subspace of $\mathbb{R}^n$.

4. (a) Let $S = \{[1,-1,1], [2,-3,3], [0,1,-1]\}$ be a subset of $\mathbb{R}^3$. Using Simplified Span Method, find a simplified general form for all vectors in span S.

(b) Use Independent Test Method, to determine whether the following subset of $\mathbb{R}^4$
$S = \{[1, 2, -1, 1], [2, 1, 0, 1], [2, -2, 1, 0], [11, 1, 1, 4]\}$ is linearly independent.

(c) Define dimension of a vector space V. Specify the standard basis for the following vector spaces and determine which of following vector spaces is finite dimensional or infinite dimensional

(i) vector space $P_n(x)$ of all polynomials with degree atmost n.

(ii) vector space P(x) of real polynomials.

5. (a) Prove that the function $f: M_{nn} \to M_{nn}$ given by $f(A) = A - A^t$, where A^t denotes the transpose of the matrix A and M_{nn} is the vector space of all $n \times n$ matrices is a linear operator.